

7. **(C)** On a problem like this, it can be helpful to look ahead to the answers to have a sense of what the answer might look like. It appears we want a solution that has only the x values on either the numerator or denominator and only the y values on the opposite side. With that in mind, simplify the expression:

$$\frac{x^a y^{-b}}{x^{-a} y^b} \rightarrow$$

$$\frac{x^a x^a}{y^b y^b} \rightarrow$$

$$\frac{x^{2a}}{y^{2b}}$$

8. **(3)**

$$\sqrt[3]{125} + \sqrt[3]{64} = x^2 \rightarrow$$

$$5 + 4 = 9 = x^2 \rightarrow 3 = x$$

9. **(25)**

$$x^{-2} y^{-1} = \frac{1}{5} \rightarrow$$

Flip both sides $\rightarrow$

$$x^2 y = 5 \rightarrow$$

Square both sides to make it the desired expression $\rightarrow$

$$x^4 y^2 = 25$$

10. **(2)**

$$\sqrt{\frac{x}{2}} = 4 \rightarrow$$

Square both sides $\rightarrow$

$$\frac{x}{2} = 16 \rightarrow$$

$$x = 32$$

Take the fifth root of 32 $\rightarrow$

$$\sqrt[5]{32} = 2$$

Advanced Math—Solving Quadratic Functions

Factoring Quadratic Equations

If a quadratic function can be easily factored to identify the solutions, use factoring to solve.

➤ Example 1

$$x^2 - 9x + 20 = 0$$

What are the solutions for x in the equation above?

✓ Solution

Factor the expression to solve for the solutions.

$$\begin{aligned}x^2 - 9x + 20 &= 0 \rightarrow \\(x - 4)(x - 5) &= 0\end{aligned}$$

With the equation in this form, you can identify which values of x will make the expression zero:

$$\begin{aligned}x - 4 &= 0 \rightarrow x = 4 \\ \text{or} \\ x - 5 &= 0 \rightarrow x = 5\end{aligned}$$

So, x could equal 4 or 5.

➤ Example 2

$$3x^2 - 3x = 6$$

What are the solutions for x in the equation above?

✓ Solution

Even though this equation is not as obviously factorable as the previous example, with a little bit of manipulation you can get it in a factorable form:

$$3x^2 - 3x = 6 \rightarrow$$

Divide both sides by 3 $\rightarrow$

$$x^2 - x = 2 \rightarrow$$

$$x^2 - x - 2 = 0$$

Factor the expression $\rightarrow$

$$(x - 2)(x + 1) = 0$$

Based on this, the two values that would be solutions are 2 and -1 , since either value if plugged in for x would make the entire expression equal to zero.

Practice

Use factoring to find the solutions to the equations.

$$x^2 - 6x + 8 = 0$$

$$x^2 - 10x + 21 = 0$$

$$x^2 - 6x - 16 = 0$$

$$x^3 - 4x = 0$$

$$x^3 + 5x^2 + 6x = 0$$

Solutions

$$x^2 - 6x + 8 = 0 \rightarrow$$

$$(x - 4)(x - 2) = 0 \rightarrow$$

$$x = 4 \text{ or } x = 2$$

$$x^2 - 10x + 21 = 0 \rightarrow$$

$$(x - 7)(x - 3) = 0 \rightarrow$$

$$x = 7 \text{ or } x = 3$$

$$x^2 - 6x - 16 = 0 \rightarrow$$

$$(x - 8)(x + 2) = 0 \rightarrow$$

Solutions: 8, -2

$$x^3 - 4x = 0 \rightarrow$$

$$x(x^2 - 4) = 0 \rightarrow$$

$$x(x + 2)(x - 2) = 0$$

Solutions are 0, -2, or 2

$$x^3 + 5x^2 + 6x = 0 \rightarrow$$

$$x(x^2 + 5x + 6) = 0 \rightarrow$$

$$x(x + 3)(x + 2) = 0$$

Solutions are 0, -3, or -2

Square Root Method

If a quadratic equation does *not* have an x term, you may want to use the square root method—take the square root of both sides of the equation to arrive at the solution(s). The positive and negative cases will be solutions.

► Example

$$x^2 - 16 = 0$$

What are the solutions for x in the equation above?

✓ Solution

This equation lacks an x term, so we can use the square root method to solve:

$$\begin{aligned}x^2 - 16 &= 0 \rightarrow \\x^2 &= 16 \rightarrow \\x &= \pm 4\end{aligned}$$

Practice

Use the square root method to solve these equations.

$$\begin{aligned}x^2 &= 81 \\x^2 - 64 &= 0 \\(x - 3)^2 &= 25\end{aligned}$$

Solutions

$$\begin{aligned}x^2 &= 81 \rightarrow \\\sqrt{x^2} &= \sqrt{81} \rightarrow \\x &= \pm 9\end{aligned}$$

$$\begin{aligned}x^2 - 64 &= 0 \rightarrow \\\sqrt{y^2} &= 64 \rightarrow \\y &= \pm 8\end{aligned}$$

$$\begin{aligned}(x - 3)^2 &= 25 \rightarrow \\\sqrt{(x - 3)^2} &= \sqrt{25} \rightarrow \\x - 3 &= \pm 5\end{aligned}$$

Then, set up two equations to solve for the potential values of x :

$$x - 3 = 5 \rightarrow x = 8$$

or

$$x - 3 = -5 \rightarrow x = -2$$

So, the solutions are 8 and -2 .

Quadratic Formula

An equation with constants a , b , and c and a variable x that is written in the form $ax^2 + bx + c = 0$ may be solved using the *quadratic formula*:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula when factoring the equation looks challenging, often when the x^2 term has a constant other than 1 in front of it (examples: $6x^2$ or $\frac{1}{3}x^2$).

▸ Example

$$3x^2 + 5x - 2 = 0$$

What are the solutions for x in the equation above?

✓ Solution

The values of the constants are $a = 3$, $b = 5$, and $c = -2$. Plug these values into the quadratic formula to solve for x .

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \\x &= \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)} \rightarrow \\x &= \frac{-5 \pm \sqrt{25 + 24}}{6} \rightarrow \\x &= \frac{-5 \pm \sqrt{49}}{6} \rightarrow \\x &= -\frac{5}{6} \pm \frac{\sqrt{49}}{6} \rightarrow -\frac{5}{6} \pm \frac{7}{6}\end{aligned}$$

So, the solutions are:

$$-\frac{5}{6} + \frac{7}{6} = \frac{2}{6} = \frac{1}{3}$$

or

$$-\frac{5}{6} - \frac{7}{6} = -\frac{12}{6} = -2$$

Practice

$$1) x^2 + 5x + 2 = 0$$

$$2) x^2 - 5x - 7 = 0$$

$$3) x^2 + 3x = 2$$

$$4) x^2 + 4x - 5 = 0$$

How many real solutions does the equation $5x^2 - 3x + 2 = 0$ have?

Solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1) x^2 + 5x + 2 = 0 \rightarrow$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(2)}}{2(3)} \rightarrow$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{6} \rightarrow$$

$$x = \frac{-5 \pm 1}{6} \rightarrow x = -1 \text{ or } -\frac{2}{3}$$

$$2) x^2 - 5x - 7 = 0 \rightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-7)}}{2(2)} \rightarrow$$

$$x = \frac{5 \pm \sqrt{25 + 56}}{4} \rightarrow$$

$$x = \frac{5 \pm \sqrt{81}}{4} \rightarrow$$

$$x = \frac{5 \pm 9}{4} \rightarrow$$

$$x = \frac{5 + 9}{4} = 3.5 \text{ or } x = \frac{5 - 9}{4} = -1$$

$$3x^2 + 3x = 2 \rightarrow$$

$$3x^2 + 3x - 2 = 0 \rightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-2)}}{2(5)} \rightarrow$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{10} \rightarrow$$

$$x = \frac{-3 \pm \sqrt{49}}{10} \rightarrow$$

$$x = \frac{-3 \pm 7}{10} \rightarrow$$

$$x = \frac{-3 + 7}{10} = \frac{4}{10} = 0.4$$

or

$$x = \frac{-3 - 7}{10} = \frac{-10}{10} = -1$$

$$3x^2 + 4x - 5 = 0 \rightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-5)}}{2(3)} \rightarrow$$

$$x = \frac{-4 \pm \sqrt{16 + 60}}{6} \rightarrow$$

$$x = \frac{-4 \pm \sqrt{76}}{6} \rightarrow$$

simplify the $\sqrt{76}$ by factoring out the square root of 4 $\rightarrow$

$$x = \frac{-4 \pm \sqrt{4(19)}}{6} \rightarrow \frac{-4 \pm 2\sqrt{19}}{6} \rightarrow \frac{-2 \pm \sqrt{19}}{3}$$

so, x can be:

$$-\frac{2}{3} + \frac{\sqrt{19}}{3} \text{ or } -\frac{2}{3} - \frac{\sqrt{19}}{3}$$

To see how many real solutions there are, find the value of the *discriminant*, i.e., the component of the quadratic formula underneath the square root sign: $b^2 - 4ac$. If it is positive, there are two real solutions. If it is zero, there is just one real solution. And if it is negative, there are no real solutions since you would have to take the square root of a negative number.

In the equation $5x^2 - 3x + 2 = 0$, $b^2 - 4ac = (-3)^2 - 4(5)(2) = 9 - 40 = -31$.

So, there are *zero* real solutions to the equation.

Completing the Square

Use completing the square particularly to work with problems involving parabolas. Completing the square will enable you to more easily identify key information about the parabola. (Note—parabolas will be covered later in the chapter.)

► Example

$$x^2 + 8x = 20$$

What are the values for x in the above equation?

✓ Solution

Here is how you can use *completing the square* to solve this equation.

First, take half of 8 (which is 4), square it, and add it to both sides of the equation.

$$\begin{aligned}x^2 + 8x &= 20 \rightarrow \\x^2 + 8x + 16 &= 20 + 16 \rightarrow \\x^2 + 8x + 16 &= 36\end{aligned}$$

Now, you can factor the left-hand side in a squared form:

$$(x + 4)^2 = 36$$

Then, take the square root of both sides:

$$\begin{aligned}\sqrt{(x + 4)^2} &= \sqrt{36} \rightarrow \\x + 4 &= \pm 6\end{aligned}$$

Finally, solve for x :

$$x + 4 = 6 \rightarrow x = 6 - 4 = 2$$

or

$$x + 4 = -6 \rightarrow x = -6 - 4 = -10$$

So, x could be either 2 or -10 .

Practice

Solve each of these using completing the square.

$$x^2 + 6x = -8$$

$$x^2 + 8x = 9$$

$$x^2 + 3x = 1$$

Solutions

$$x^2 + 6x = -8 \rightarrow$$

$$x^2 + 6x + 9 = -8 + 9 \rightarrow$$

$$(x + 3)^2 = 1 \rightarrow$$

$$x + 3 = \pm 1$$

So, x could equal -4 or -2 .

$$x^2 + 8x = 9 \rightarrow$$

$$x^2 + 8x + 16 = 9 + 16 \rightarrow$$

$$(x + 4)^2 = 25 \rightarrow$$

$$x + 4 = \pm 5$$

So, x could equal 1 or -9 .

$$4x^2 + 3x = 1 \rightarrow$$

$$x^2 + \frac{3}{4}x = \frac{1}{4} \rightarrow$$

Take half of $\frac{3}{4}$, square it, and add to both sides:

$$x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = \frac{1}{4} + \left(\frac{3}{8}\right)^2 \rightarrow$$

$$x^2 + \frac{3}{4}x + \frac{9}{64} = \frac{1}{4} + \frac{9}{64} \rightarrow \frac{16}{64} + \frac{9}{64} = \frac{25}{64} \rightarrow$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{25}{64} \rightarrow \sqrt{\frac{25}{64}} = + / - \frac{5}{8}$$

So, x could equal:

$$x + \frac{3}{8} = \frac{5}{8} \rightarrow x = \frac{2}{8} = \frac{1}{4}$$

or

$$x + \frac{3}{8} = -\frac{5}{8} \rightarrow x = -\frac{8}{8} = -1$$

Plugging in the Answers

Try plugging in the answer choices in a quadratic equation if solving the problem algebraically will take longer than working backwards from the four options.

► Example

$$\frac{1}{x+2} + \frac{3}{2x+4} = \frac{1}{2}$$

What is the value of x in the equation above?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

✓ Solution

While this problem could be solved algebraically, it will likely save you time to work backwards from the answer options. Since the answers are in order from least to greatest, start with one of the middle options to minimize the potential trial and error.

Start with 2:

$$\begin{aligned}\frac{1}{x+2} + \frac{3}{2x+4} &= \frac{1}{2} \rightarrow \\ \frac{1}{2+2} + \frac{3}{2(2)+4} &= \frac{1}{2} \rightarrow \\ \frac{1}{4} + \frac{3}{8} &= \frac{5}{8} \neq \frac{1}{2}\end{aligned}$$

This option did not work. Since the sum was larger than $\frac{1}{2}$, we should try another answer choice that will make the expression have a smaller fraction—this comes from having a larger denominator. So, we will next try choice (C), 3:

$$\frac{1}{x+2} + \frac{3}{2x+4} = \frac{1}{2} \rightarrow$$

$$\frac{1}{3+2} + \frac{3}{2(3)+4} = \frac{1}{2} \rightarrow$$

$$\frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$$

This checks out, so choice **(C)**, 3, is correct.

Practice

Use plugging in the answers to solve.

$$1(x-2)^2 + 2 = 2x$$

Which of the following is a value of x ?

- A) 1
- B) 2
- C) 3
- D) 4

$$\frac{1}{x+2} + \frac{1}{x+3} = \frac{3}{2}$$

Which of the following is a value of x ?

- A) -2
- B) -1
- C) 0
- D) 1

Which of the following is/are solutions for x in the following equation?

$$3x^2 - 7x = x^3$$

- I. Zero
- II. One
- III. Two

- A) I only
- B) II only
- C) I and II only
- D) All of the above

Solutions

1. $4(x - 2)^2 + 2 = 2x \rightarrow$
 $4(3 - 2)^2 + 2 = 2(3) \rightarrow$
 $4(1)^2 + 2 = 6 \rightarrow$
 $6 = 6$

So, choice **(C)**, 3, works.

2. $\frac{1}{x+2} + \frac{1}{x+3} = \frac{3}{2} \rightarrow$
 $\frac{1}{-1+2} + \frac{1}{-1+3} = \frac{3}{2} \rightarrow$
 $\frac{1}{1} + \frac{1}{2} = \frac{3}{2}$

So, choice **(B)**, -1, works.

3. First, let's try zero:
 $8x^2 - 7x = x^3 \rightarrow$
 $8(0)^2 - 7(0) = 0^3 \rightarrow$
 $0 = 0$

So, option I, zero, does work.

Next, let's try 1:
 $8x^2 - 7x = x^3 \rightarrow$
 $8(1)^2 - 7(1) = 1^3 \rightarrow$
 $8 - 7 = 1 \rightarrow$
 $1 = 1$

So option II, 1, does work.

Finally, let's try 2:

$$\begin{aligned}
 8x^2 - 7x &= x^3 \rightarrow \\
 8(2)^2 - 7(2) &= 2^3 \rightarrow \\
 32 - 14 &= 8 \rightarrow \\
 18 &\neq 8
 \end{aligned}$$

So option III, 2, does NOT work. Therefore, the correct answer is choice (C). You could also approach this in an efficient way by realizing that if option III DOES work, choice (D) will have to be the answer since it is the only option with option III. Further, if you are evaluating quadratics on the calculator section, you may want to graph the function to more quickly identify its roots.

Undefined Functions

When a function is divided by zero, it may be undefined—that is, it will have no solutions. If you were to divide a number like 3 by 0, it would be undefined since it is impossible to divide 3 into zero parts.

➤ Example

When is this function undefined?

$$f(x) = \frac{x^3 - x^2 + 2}{x - 4}$$

✓ Solution

Determine the value of x that would cause the function to have zero in the denominator. Set the denominator equal to zero and solve for x :

$$x - 4 = 0 \rightarrow x = 4$$

Therefore, the function is undefined when x is equal to 4.

Practice

Find the values of x that will make the expression undefined.

$$\frac{5}{0}$$

$$\frac{2x+1}{x+4}$$

$$\frac{7}{x^2-9}$$

Solutions

$$\frac{5}{0} \rightarrow \frac{5}{0}$$

So, the function is undefined when x is 0.

$$\frac{2x+1}{x+4} \rightarrow \frac{12x+1}{0} \rightarrow$$

$$x+4=0 \rightarrow x=-4$$

So, the function is undefined when x is -4 .

$$\frac{7}{x^2-9} \rightarrow \frac{7}{0} \rightarrow$$

$$x^2-9=0 \rightarrow$$

$$x^2=9 \rightarrow x=\pm 3$$

So, the function is undefined when x is 3 or -3 .

Extraneous Solutions

When dealing with equations involving roots, it is a good idea to test the solutions to see if they are *extraneous*—meaning, they do not work in the original equation. The reason they typically do not work is that they would require taking the square root of a negative number, which would result in an imaginary solution.

► Example

$$\sqrt{x-1} = x-7$$

What are the solutions for x in the equation above?

✓ Solution

Start by squaring both sides to get rid of the radical sign:

$$\begin{aligned}\sqrt{x-1} &= x-7 \rightarrow \\ x-1 &= x^2-14x+49\end{aligned}$$

Then, simplify the equation and factor to find the potential solutions:

$$\begin{aligned}x-1 &= x^2-14x+49 \rightarrow \\ x^2-15x+50 &= 0 \rightarrow \\ (x-10)(x-5) &= 0\end{aligned}$$

So, the potential solutions are 10 and 5. Now, to be sure we don't have any extraneous solutions, plug 10 and 5 back in to the original equation to see if they work:

$$\begin{aligned}\sqrt{x-1} &= x-7 \rightarrow \\ \sqrt{10-1} &= 10-7 \rightarrow \\ \sqrt{9} &= 3\end{aligned}$$

So, 10 works in the original equation. Now let's try 5.

$$\begin{aligned}\sqrt{5-1} &= 5-7 \rightarrow \\ \sqrt{4} &= 5-7 \rightarrow \\ \sqrt{4} &\neq -2\end{aligned}$$

The square root of 4 cannot be equal to -2 in this case, since we would need an imaginary number to make it work. Therefore, 5 is an extraneous solution and the only solution is 10.

Practice

Solve each of these equations for x , watching out for extraneous solutions.

$$c = \sqrt{12-x}$$

$$\sqrt{x+1} + 5 = x$$

$$\sqrt{x-6} = x-8$$

Solutions

$$c = \sqrt{12-x} \rightarrow$$

$$c^2 = 12-x \rightarrow$$

$$c^2 + x - 12 = 0 \rightarrow$$

$$(x+4)(x-3)$$

So, -4 and 3 are potential solutions. Check to see if either is extraneous.

$$-4 = \sqrt{12 - (-4)} \rightarrow$$

$$-4 \neq 4$$

So, -4 does NOT work.

$$3 = \sqrt{12-3} \rightarrow$$

$$3 = 3$$

So, 3 does work.

$$\sqrt{x+1} + 5 = x \rightarrow$$

$$\sqrt{x+1} = x-5 \rightarrow$$

$$c+1 = x^2 - 10x + 25 \rightarrow$$

$$1 = x^2 - 11x + 24 \rightarrow$$

$$1 = (x-8)(x-3)$$

8 and 3 are potential solutions. Check to see if either is extraneous.

$$\sqrt{8+1} + 5 = 8 \rightarrow$$

$$3 + 5 = 8 \rightarrow$$

$$8 = 8$$

8 does work.

$$\sqrt{3+1} + 5 = 3 \rightarrow$$

$$\sqrt{4} + 5 = 3 \rightarrow$$

$$2 + 5 \neq 3$$

3 does NOT work.

$$\sqrt{x-6} = x-8 \rightarrow$$

$$x-6 = (x-8)^2 \rightarrow$$

$$x-6 = x^2 - 16x + 64 \rightarrow$$

$$0 = x^2 - 17x + 70 \rightarrow$$

$$0 = (x-10)(x-7)$$

10 and 7 are potential solutions. Check to see if either is extraneous.

$$\sqrt{x-6} = x-8 \rightarrow$$

$$\sqrt{10-6} = 10-8 \rightarrow$$

$$\sqrt{4} = 2$$

So, 10 does work.

$$\sqrt{x-6} = x-8 \rightarrow$$

$$\sqrt{7-6} = 7-8 \rightarrow$$

$$\sqrt{1} = 1 \neq -1$$

So, 7 does NOT work.

Synthetic Division

On occasion, the SAT will ask you to divide a polynomial. The easiest way to do this is using synthetic division.

► Example 1

What is the result when $2x^2 + 5x - 3$ is divided by $x - 4$?

✓ Solution

Set up the problem by taking the coefficients of the terms of the polynomial and placing the numerical term of the divisor to the left of them like this:

$$\begin{array}{r|rrr} 4 & 2 & 5 & -3 \end{array}$$

Next, bring down each of the coefficients, multiplying the columns one-by-one by the divisor 4, and create sums to see what the divided polynomial and remainder would be:

$$\begin{array}{r|rrr} 4 & 2 & 5 & -3 \\ & & 8 & 52 \\ \hline & 2 & 13 & 49 \end{array}$$

The answer would be $2x + 13$ with a remainder of $\frac{49}{x-4}$.

➤ Example 2

Based on the previous problem, is $x - 4$ a factor of $2x^2 + 5x - 3$?

✓ Solution

For an expression to be a factor of the polynomial, you must be able to divide the polynomial by that expression and NOT have a remainder, i.e., the remainder must be zero. Since there is a remainder of $\frac{49}{x-4}$ when the expression is divided, $x - 4$ is NOT a factor of $2x^2 + 5x - 3$.

Practice

- 1A. What is the result when $x^2 + 2x - 15$ is divided by $x + 5$?
- 1B. Is $x + 5$ a factor of $x^2 + 2x - 15$?
- 2A. What is the result when $4x^2 - 3x - 5$ is divided by $x - 3$?
- 2B. Is $x - 3$ a factor of $4x^2 - 3x - 5$?

Solutions

1A.

$$\begin{array}{r|rrr} -5 & 1 & 2 & -15 \\ & & -5 & 15 \\ \hline & 1 & -3 & 0 \end{array}$$

Therefore, the answer is $x - 3$.

1B. Since there was no remainder when it was divided, this is a factor of the expression.

2A.

$$\begin{array}{r|rrr} 3 & 4 & -3 & -5 \\ & & 12 & 27 \\ \hline & 4 & 9 & 22 \end{array}$$

So, the result is $4x + 9$ with a remainder of $\frac{22}{x-3}$.

2B. Since there was a remainder when it was divided, this is NOT a factor of the expression.

SAT Math Strategy

Many SAT quadratic function problems can be solved in different ways. Use whatever approach is easiest for you.

► Example

If $2x^2 - 6x = 20$, what are the solutions for x ?

- (A) -2 or -5
- (B) -2 or 5
- (C) 2 or -5
- (D) 2 or 5

✓ Solution

Method 1: Factoring. If you notice that all the numbers in the problem have 2 as a factor, you can take a 2 out of everything to simplify. Then factoring will probably appear to be the easiest method.

$$\begin{aligned} 2x^2 - 6x &= 20 \rightarrow \\ 2x^2 - 6x - 20 &= 0 \rightarrow \\ x^2 - 3x - 10 &= 0 \rightarrow \\ (x + 2)(x - 5) &= 0 \end{aligned}$$

The correct answer is therefore **(B)**, -2 or 5.

Method 2: Quadratic Formula. If you do not see an easy way to factor the equation, you can put the equation in standard form and use the quadratic formula to solve. This can be a good approach if you are quick and careful with your calculations.

$$\begin{aligned} 2x^2 - 6x &= 20 \rightarrow \\ 2x^2 - 6x - 20 &= 0 \rightarrow \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \\ x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-20)}}{2(2)} \rightarrow \\ x &= \frac{6 \pm \sqrt{36 + 160}}{4} \rightarrow \\ x &= \frac{6 \pm \sqrt{196}}{4} \rightarrow \\ x &= \frac{6 \pm 14}{4} \end{aligned}$$

So, x could be either $\frac{6 + 14}{4} = \frac{20}{4} = 5$ or $\frac{6 - 14}{4} = \frac{-8}{4} = -2$

Method 3: Plug in the Answers. Look ahead to the answer choices to notice that all of them are variations of ± 2 and ± 5 . So, you essentially just have four numbers to plug in to the original equation.

$$\begin{aligned} 2(-2)^2 - 6(-2) &= 20 \rightarrow \\ 8 + 12 &= 20 \\ -2 &\text{ works.} \end{aligned}$$

$$2(2)^2 - 6(2) = 20 \rightarrow$$

$$8 - 12 \neq 20$$

2 does NOT work.

$$2(-5)^2 - 6(-5) = 20 \rightarrow$$

$$2(25) + 30 = 20 \rightarrow$$

$$50 + 30 \neq 20$$

-5 does NOT work.

$$2(5)^2 - 6(5) = 20 \rightarrow$$

$$2(25) - 30 = 20 \rightarrow$$

$$50 - 30 = 20$$

5 does work.

If there were more numbers to plug in, this approach would have been more difficult.

On school tests, you can often get in a rhythm with the questions you are solving. Each question on a test or quiz may be a variation of “solve for x .” On the SAT Math, be careful on quadratic function questions to first understand what the question is asking, then start working on the solution.

SAT Math Strategy

Do not go on autopilot with the questions—carefully consider what you are asked to do.

► Example

Variables x and y are related to each other in a quadratic function of the form $y = kx^2$, in which k is a constant. If y is 12 when x is 2, and y is 48 when x is 4, what is the value of y when x is 5?

(A) 30

(B) 45

(C) 60

(D) 75

Solution

This is not a problem that can be solved in a typical fashion, like using the quadratic formula or factoring. We must fully understand what the question tells us.

- The variables are related to each other in an equation like $y = kx^2$.
- x and y are variables while k is a constant.
- We have ordered pairs of values—(2, 12) and (4, 48)—that we can plug in to see what the constant k would be.

Given that information, let's plug in the ordered pair (2, 12) into the equation, since it is smaller and easier to work with than (4, 48):

$$\begin{aligned}y &= kx^2 \rightarrow \\12 &= k(2)^2 \rightarrow \\12 &= 4k \rightarrow \\k &= 3\end{aligned}$$

So, the function in the question is $y = 3x^2$. We now can see what the value of y is when x is 5:

$$\begin{aligned}y &= 3x^2 \rightarrow \\y &= 3(5)^2 \rightarrow \\y &= 3(25) = 75\end{aligned}$$

The answer is therefore **(D)**, 75.

Practice Drill #1—Solving Advanced Equations

(If timing, take about 12 minutes to complete.)

For what positive value of x is this function undefined?

$$f(x) = \frac{1}{x^2 - 1}$$

A) 1

- B) 2
- C) 3
- D) 4

If $x^2 - 3x - 10 = 0$, for what value of x is this equation true given that $x > 0$?

- A) 3
- B) 4
- C) 5
- D) 6

Which of the following would be a solution to this equation?

$$21 - \sqrt{x} = 10$$

- A) 11
- B) 18
- C) 55
- D) 121

Which of the following provides the two solutions to this equation?

$$3x^2 + 5x + 2 = 0$$

- A) $-\frac{2}{3}$ or -1
- B) $\frac{1}{6}$ or $\frac{2}{5}$
- C) $-\frac{3}{4}$ or 7
- D) $\frac{6}{11}$ or -8

$$x^2 - 15 = x + 5$$

What is the product of the solutions to the above equation?

- A) -35

B) -20

C) 1

D) 9

$$\frac{x^2 - y^2}{3} = 0$$

What value of y would always make the above equation true?

A) x

B) x^2

C) $4x$

D) $8x$

Fill-In Practice: Write your answer in the underlined blank under each question.

If $\frac{(x+2)}{(x-3)} - 2 = \frac{7}{(x-3)}$, what is the solution for x ?

Answer: _____

If $2\sqrt{x} + 1 = 10 - \sqrt{x}$, what is the solution for x ?

Answer: _____

$$(x+b)(x+b) = x^2 + 4x + 4$$

If the above equation has infinitely many solutions, what is the value of the constant b ?

Answer: _____

If $\frac{6x^2 - 5x + 8}{2} = x^2 - 3$, how many real solutions are there for x ?

Answer: _____

Solutions

1. **(A)** For the function $f(x) = \frac{1}{x^2 - 1}$ to be undefined, we need a value of x that will make the denominator equal to zero. So, set up an equation to solve for the positive value of x needed:

$$\begin{aligned}x^2 - 1 &= 0 \rightarrow \\(x + 1)(x - 1) &= 0\end{aligned}$$

So, x could equal 1 or -1 . Since we need a positive value of x , the correct answer is 1.

2. **(C)** Solve this by factoring:

$$\begin{aligned}x^2 - 3x - 10 &= 0 \rightarrow \\(x - 5)(x + 2) &= 0\end{aligned}$$

So, x could be either 5 or -2 . Since the answer must be positive, the correct answer is 5.

3. **(D)** Use the square root method to solve:

$$\begin{aligned}21 - \sqrt{x} &= 10 \rightarrow \\11 - \sqrt{x} &= 0 \rightarrow \\11 &= \sqrt{x} \rightarrow \\121 &= x\end{aligned}$$

4. **(A)** Given that there is a 3 in front of the x^2 term, the quadratic formula will be the best way to solve this:

$$3x^2 + 5x + 2 = 0 \rightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(2)}}{2(3)} \rightarrow$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{6} \rightarrow$$

$$x = \frac{-5 \pm 1}{6} \rightarrow$$

$$x = -\frac{5}{6} + \frac{1}{6} = -\frac{4}{6} = -\frac{2}{3}$$

and

$$x = -\frac{5}{6} - \frac{1}{6} = -\frac{6}{6} = -1$$

5. **(B)** Simplify and factor the equation to determine the two solutions, then multiply them together to find their product:

$$x^2 - 15 = x + 5 \rightarrow$$

$$x^2 - x - 20 = 0 \rightarrow$$

$$(x - 5)(x + 4) = 0$$

The solutions are therefore 5 or -4 . The product of them is $5 \times (-4) = -20$.

6. **(A)** Solve for y by simplifying the equation, then using the square root method:

$$\frac{x^2 - y^2}{3} = 0 \rightarrow$$

$$x^2 - y^2 = 0 \rightarrow$$

$$x^2 = y^2 \rightarrow$$

$$\sqrt{x^2} = \sqrt{y^2} \rightarrow$$

$$x = y$$

7. **(1)** Multiply the equation by $(x - 3)$ to remove the $(x - 3)$ from the denominator:

$$\begin{aligned}(x - 3) \times \left(\frac{(x + 2)}{(x - 3)} - 2 \right) &= (x - 3) \times \left(\frac{7}{(x - 3)} \right) \rightarrow \\(x + 2) - 2(x - 3) &= 7 \rightarrow \\x + 2 - 2x + 6 &= 7 \rightarrow \\-x + 8 &= 7 \rightarrow \\-x &= -1 \rightarrow x = 1\end{aligned}$$

8. **(9)** Simplify and use the square root method:

$$\begin{aligned}2\sqrt{x} + 1 &= 10 - \sqrt{x} \rightarrow \\3\sqrt{x} &= 9 \rightarrow \\\sqrt{x} &= 3 \rightarrow \\(\sqrt{x})^2 &= 3^2 \rightarrow \\x &= 9\end{aligned}$$

9. **(2)** For there to be infinitely many solutions, the equation must be equivalent on both sides. FOIL the left-hand side to see what the value of b must be for this to be true.

$$\begin{aligned}(x + b)(x + b) &= x^2 + 4x + 4 \rightarrow \\x^2 + 2bx + b^2 &= x^2 + 4x + 4\end{aligned}$$

If b is equal to 2, both sides are equivalent, giving us infinitely many solutions.

10. **(0)** Simplify this equation to get it in standard form.

$$\begin{aligned}\frac{6x^2 - 5x + 8}{2} &= x^2 - 3 \rightarrow \\6x^2 - 5x + 8 &= 2x^2 - 6 \rightarrow \\4x^2 - 5x + 14 &= 0\end{aligned}$$

Now, use the quadratic formula to solve:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \\x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(14)}}{2(4)} \rightarrow \\x &= \frac{5 \pm \sqrt{25 - 224}}{8}\end{aligned}$$

Since the discriminant (the $\sqrt{25 - 224}$ term) is negative, there will be 0 real solutions.

Practice Drill #2—Solving Advanced Equations

(If timing, take about 12 minutes to complete.)

If $x^2 - 14x + 49 = 0$, what is the value of x ?

- A) 3
- B) 5
- C) 7
- D) 9

If $n^2 - 6n + 9 = 25$, which of the following is a possible value of $n - 3$?

- A) 4
- B) 5
- C) 6
- D) 9

If $\frac{3x}{6-x} = x$, what are the value(s) of x that would satisfy the equation?

- A) 1 and -6
- B) 2 and -3
- C) 3 and 0

D) 6 and 4

What is the largest value of n that satisfies the following equation?

$$\frac{(n+4)(n-6)}{2} = 0$$

A) -4

B) -6

C) 4

D) 6

$$f(x) = \frac{2x - 13}{2x^2 - 16x + 32}$$

For what value of x is the above function $f(x)$ undefined?

A) -8

B) -4

C) 2

D) 4

What are the solutions to the equation $5x^2 - 7x + 1 = 0$?

A) $\frac{5 \pm \sqrt{23}}{14}$

B) $\frac{7 \pm \sqrt{29}}{10}$

C) $\frac{3 \pm \sqrt{37}}{11}$

D) $\frac{12 \pm \sqrt{43}}{7}$

$$\sqrt{x+4} = -4$$

What is the set of real solutions for the above equation?

A) 2 only

B) 2 and -4

- C) 4 and -2
D) There are no real solutions.

What is the remainder when $2x^2 + 3x - 1$ is divided by $x - 4$?

- A) $\frac{43}{x - 4}$
B) $\frac{22}{x - 4}$
C) $\frac{16}{x - 1}$
D) 0

If $\sqrt{x + 5} = 4$, what is the value of x ?

- A) -9
B) -5
C) 2
D) 11

$$\frac{x^2 - 27}{3x - 9} = 5$$

What value of x is a solution to the equation above?

- A) 2
B) 5
C) 6
D) 18

Solutions

1. (C) Solve by factoring:

$$x^2 - 14x + 49 = 0 \rightarrow$$

$$(x - 7)(x - 7) = 0$$

So, 7 is the only solution.

2. **(B)** Factor the expression, then use the square root method:

$$\begin{aligned}n^2 - 6n + 9 &= 25 \rightarrow \\(n - 3)(n - 3) &= 25 \rightarrow \\\sqrt{(n - 3)^2} &= \sqrt{25} \rightarrow \\n - 3 &= 5\end{aligned}$$

3. **(C)** Plugging and chugging would be a viable approach on this problem, but you can also solve it fairly easily by cross multiplying and then solving for x .

$$\begin{aligned}\frac{3x}{6 - x} &= x \rightarrow \\3x &= x(6 - x) \rightarrow \\3x &= 6x - x^2 \rightarrow \\x^2 - 3x &= 0 \rightarrow \\x^2 &= 3x \rightarrow\end{aligned}$$

So zero is a solution.

Also, 3 is a solution because

$$x^2 = 3x \rightarrow x = 3.$$

4. **(D)** This is easiest to solve by plugging in the answers. Start with the largest choice and work your way smaller if needed:

$$\begin{aligned}\frac{(n + 4)(n - 6)}{2} &= 0 \rightarrow \\\frac{(6 + 4)(6 - 6)}{2} &= 0 \rightarrow \\\frac{10 \times 0}{2} &= 0\end{aligned}$$

So, 6 works. Alternatively, keep in mind that a polynomial written in factored form automatically shows what the zeroes are.

5. **(D)** For the function $f(x) = \frac{2x - 13}{2x^2 - 16x + 32}$ to be undefined, the denominator must be zero. So, set up an equation in which just the denominator is equal to zero.

$$2x^2 - 16x + 32 = 0 \rightarrow$$

$$x^2 - 8x + 16 = 0 \rightarrow$$

$$(x - 4)(x - 4) = 0$$

So, if x equals 4, the denominator will equal zero and the function will be undefined.

6. **(B)** Use the quadratic formula to solve this equation.

$$5x^2 - 7x + 1 = 0 \rightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(1)}}{2(5)} \rightarrow$$

$$x = \frac{7 \pm \sqrt{49 - 20}}{10} \rightarrow$$

$$x = \frac{7 \pm \sqrt{29}}{10}$$

7. **(D)** Plugging in the potential answers of 2, 4, -2, and -4 is an excellent way to solve the problem. You can also solve it algebraically:

$$\sqrt{x + 4} = -4 \rightarrow$$

$$(\sqrt{x + 4})^2 = (-4)^2 \rightarrow$$

$$x + 4 = 16 \rightarrow$$

$$x = 12$$

Now, when you plug 12 back in to the original equation, you can see that it does not work:

$$\begin{aligned}\sqrt{x+4} &= -4 \rightarrow \\ \sqrt{12+4} &= -4 \rightarrow \\ 4 &\neq -4\end{aligned}$$

In fact, if you notice that in the original equation it states that the square root of $x + 4$ must equal a negative number, you can stop right there since the least that the square root of the number could be would be zero. We would need imaginary numbers to arrive at a negative solution.

8. **(A)** Use synthetic division to solve:

$$\begin{array}{r|rrr} 4 & 2 & 3 & -1 \\ & & 8 & 44 \\ \hline & 2 & 11 & 43 \end{array}$$

Take the 43 and divide it by $x - 4$ to get the remainder of $\frac{43}{x-4}$.

9. **(D)** Plugging in the answers will work well on this problem. Also, you can solve it algebraically:

$$\begin{aligned}\sqrt{x+5} &= 4 \rightarrow \\ (\sqrt{x+5})^2 &= 4^2 \rightarrow \\ x+5 &= 16 \rightarrow \\ x &= 11\end{aligned}$$

10. **(A)** You could use plugging in the answers to solve this, but you could also do it algebraically.

$$\frac{3x^2 - 27}{3x - 9} = 5 \rightarrow$$

Divide the numerator and the denominator of the left side by 3 $\rightarrow$

$$\frac{x^2 - 9}{x - 3} = 5 \rightarrow$$

$$\frac{(x + 3)(x - 3)}{(x - 3)} = 5 \rightarrow$$

$$\frac{(x + 3)\cancel{(x - 3)}}{\cancel{(x - 3)}} = 5 \rightarrow$$

$$x + 3 = 5 \rightarrow x = 2 \text{ (Note that 3 is an extraneous solution if you plug it back into the original equation.)}$$

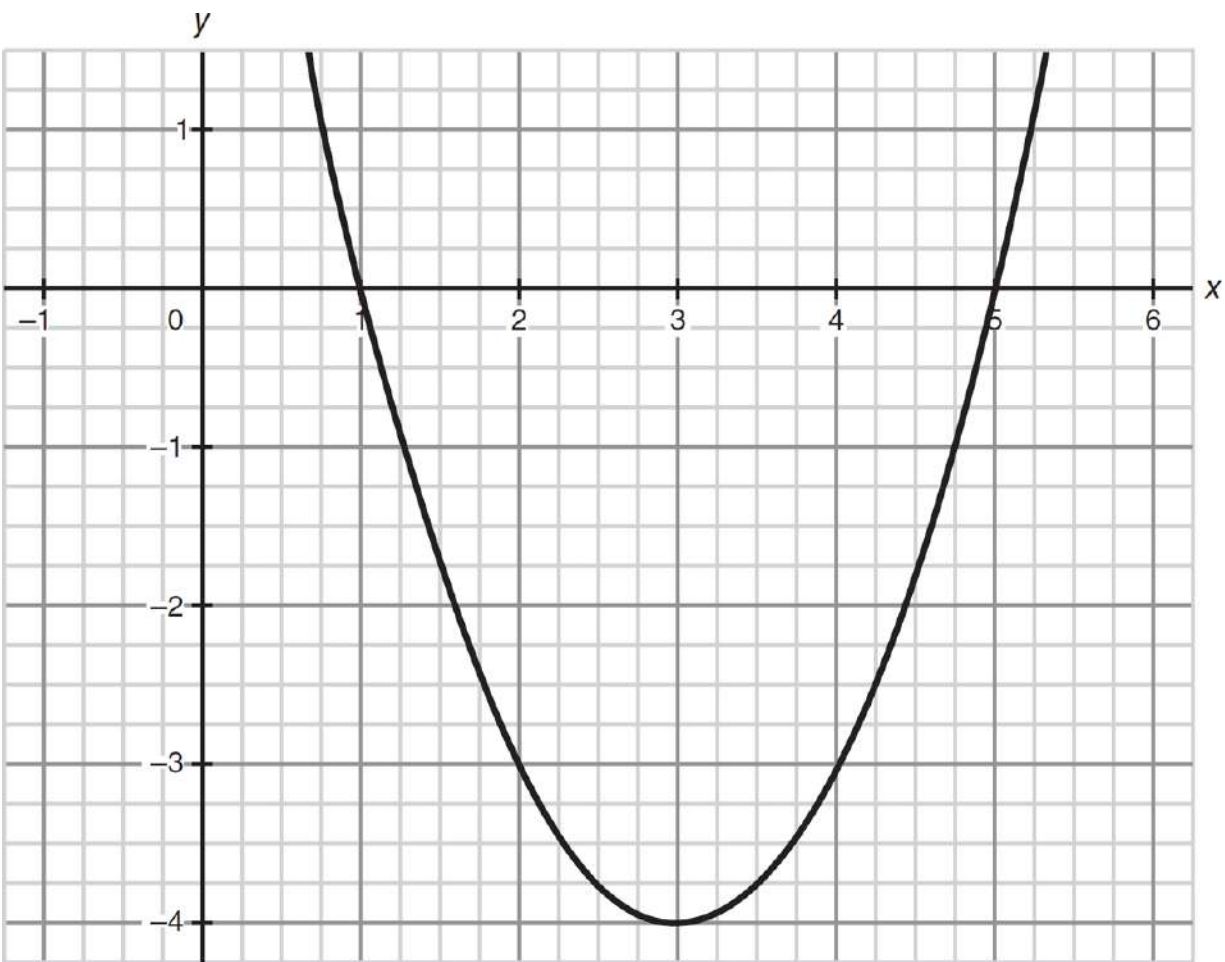
Advanced Math—Zeros, Parabolas, and Polynomial Graphing

Zeros of a Function

The *root* or *zero* of a function is the value of x for which a function like $f(x)$ has a value of zero. When an equation of the function is provided, you can identify the zeros by solving for when $f(x)$ is 0 (i.e., finding the x -intercepts or solutions). When a graph of the function is provided, you can identify the zeros by looking at the values of x for which the $f(x)$ values have coordinates at 0.

► Example

What are the zeros of the parabola of the equation $f(x) = x^2 - 6x + 5$ and with the graph below?



✓ Solution

The first way you can find the zeros is by factoring the equation:

$$f(x) = x^2 - 6x + 5 \rightarrow$$

$$f(x) = (x - 5)(x - 1)$$

So, there are zeros at x values of 5 and 1 since plugging those two values in to the function will cause the value of $f(x)$ to be 0.

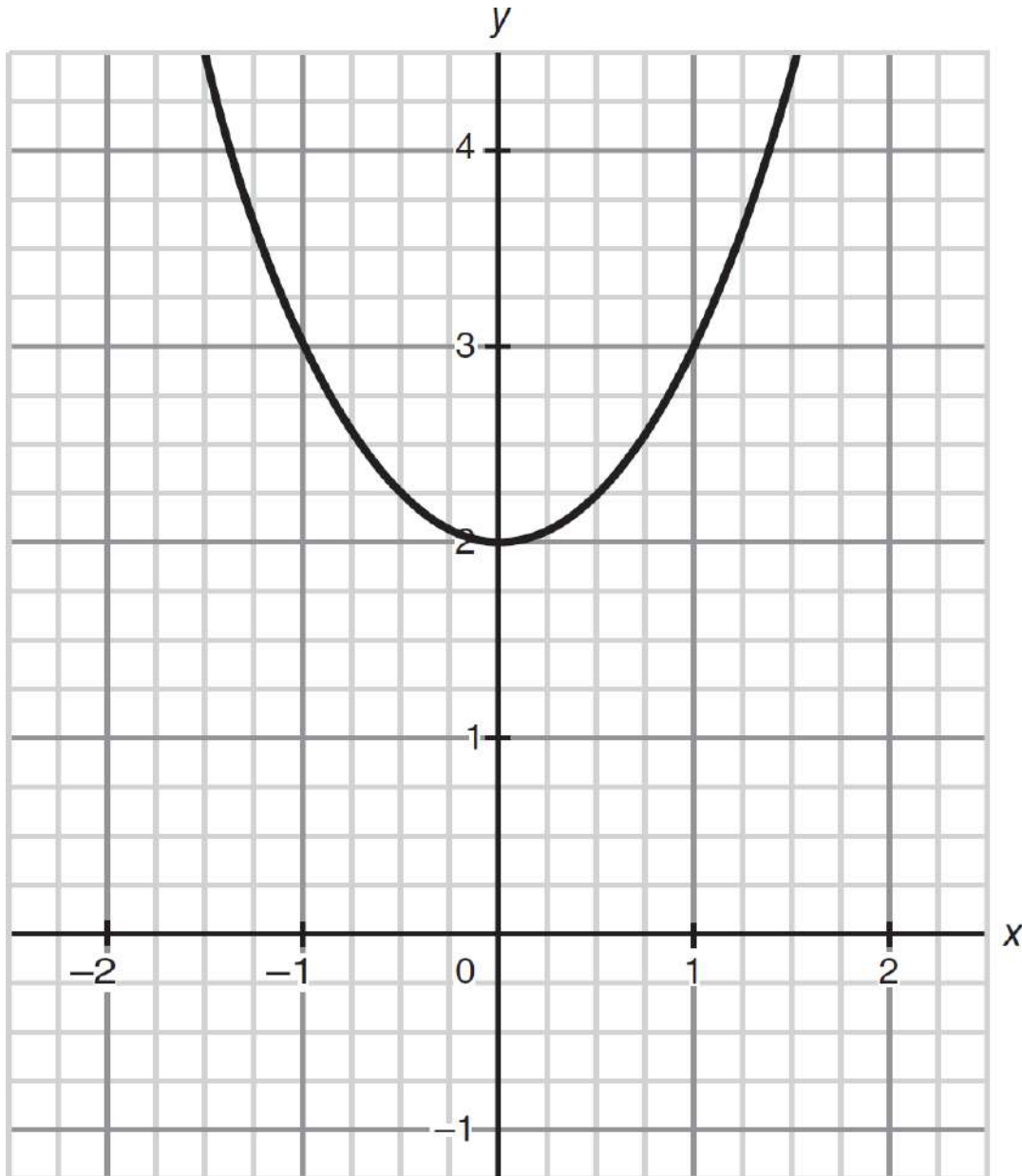
The second way you can find the zeros is by examining the graph above. Look at the points where the function intersects the x -axis, since the value of $f(x)$, or y , will be zero there. At points (1, 0) and (5, 0), the function intersects the x -axis, so 5 and 1 are zeros of the function.

Practice

What are the zeros of the function $y = (x + 3)(x - 5)$?

What are the zeros of the function $f(x) = x^2 + 4x - 12$?

What are the zeros of the function graphed below?



What are the zeros of $f(x) = x(x - 1)(x + 2)(x - 9)$?

Solutions

1. Since the function is already in factored form, it is easy to identify which values of x would make y have a value of zero. If x is either -3 or 5 , y will be zero. Therefore, the zeros are -3 and 5 .
2. Factor the equation to identify the zeros:

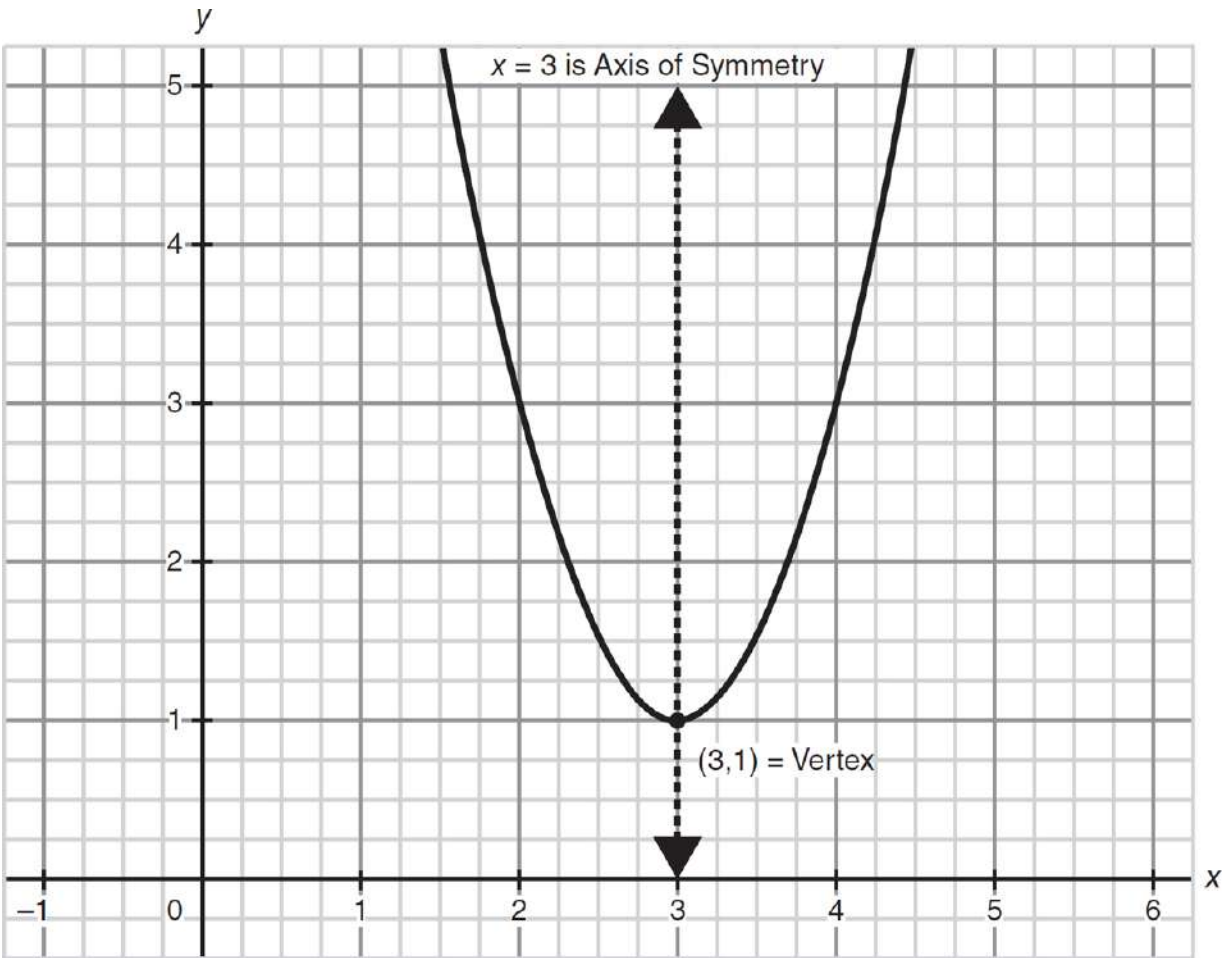
$$\begin{aligned}f(x) &= x^2 + 4x - 12 \rightarrow \\f(x) &= (x + 6)(x - 2)\end{aligned}$$

So, the zeros are at -6 and 2 .

3. Since the function does not intersect the x -axis, there are no points at which y will be zero. Therefore, there are NO real zeros in the function.
4. Examine the equation to determine the values of x that will make $f(x) = 0$. Fortunately, the equation is already factored, $f(x) = x(x - 1)(x + 2)(x - 9)$, making it easier to identify the zeros. 0 , 1 , -2 , and 9 if plugged in for x would all make the function equal to 0 , so these numbers are all zeros of the function.

Parabola Graphing

A parabola is a function with a u-shaped curve that has certain properties. Here is an example of a parabola with the equation $y = 2(x - 3)^2 + 1$:



- The vertex form of a parabola is $y = a(x - h)^2 + k$. In the above example, $y = 2(x - 3)^2 + 1$ is written in vertex form, with $a = 2$, $h = 3$, and $k = 1$.
- The value of the constant a determines the direction of the parabola—if it is positive, it opens upward, and if it is negative, it opens downward. In the example, a is $+2$, so the parabola opens upward.
- The vertex (the bottom point of the u-shape) has the coordinates (h, k) . The vertex of the parabola in the example is $(3, 1)$.
- The x -coordinate of the vertex gives the *axis of symmetry* for the parabola. The axis of symmetry in the example is $x = 3$.

The SAT Math questions involving parabolas focus heavily on identifying the vertex of a parabola.

There are four major approaches to doing this.

Method 1: Identify points h and k when the parabola is in vertex form.

> Example

What is the vertex of the parabola with the equation $y = 3(x - 4)^2 - 7$?

✓ Solution

Notice that the equation is already in vertex form, making it easy to identify the vertex:

$$\begin{aligned}y &= a(x - h)^2 + k \rightarrow \\y &= 3(x - 4)^2 - 7\end{aligned}$$

Since 4 corresponds to the h and -7 corresponds to the k , the vertex will be $(4, -7)$.

Method 2: Use the formula $x = -\frac{b}{2a}$.

When a parabola is written in the form $y = ax^2 + bx + c$, you can determine the x -coordinate of the parabola's vertex by using this formula:

$$x = -\frac{b}{2a}$$

Then, you can plug the x value back in to the original equation to solve for the y value of the vertex.

> Example

What is the vertex of the parabola with the equation $y = 4x^2 - 2x - 3$?

✓ Solution

Find the a and b values in the equation and then plug them into the formula. Constant a is 4 and constant b is -2 . Now, plug them in:

$$x = -\frac{b}{2a} \rightarrow$$

$$x = -\frac{(-2)}{2(4)} = \frac{2}{8} = \frac{1}{4}$$

Now that we know the x -coordinate of the vertex, we can plug this value back into the original equation to solve for the y -coordinate of the vertex:

$$y = 4x^2 - 2x - 3 \rightarrow$$

$$y = 4\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right) - 3 \rightarrow$$

$$y = 4\left(\frac{1}{16}\right) - \frac{1}{2} - 3 \rightarrow$$

$$y = \frac{1}{4} - \frac{1}{2} - 3 = -3\frac{1}{4}$$

So, the vertex of the parabola is $\left(\frac{1}{4}, -3\frac{1}{4}\right)$.

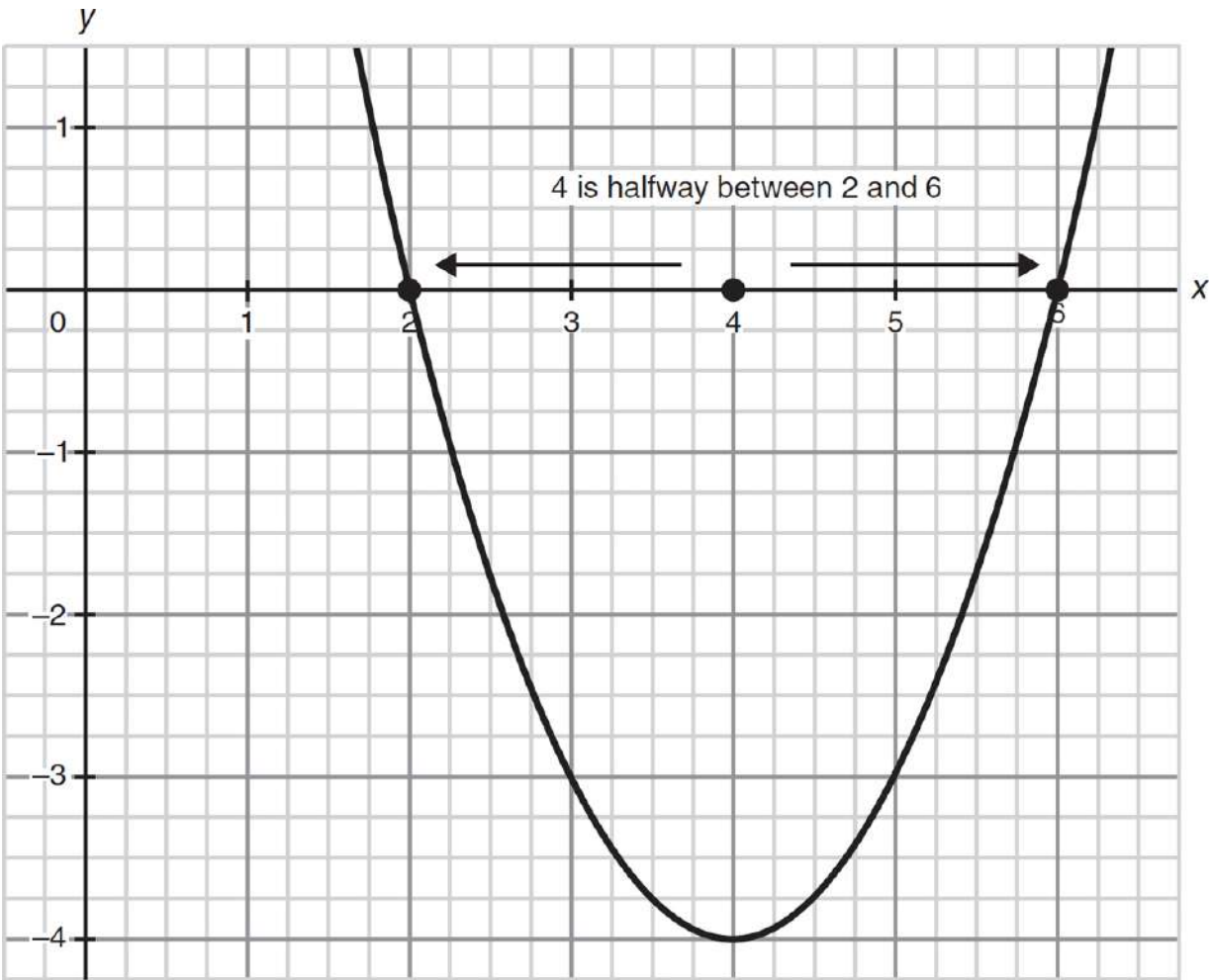
Method 3: Identify the x value halfway between the zeros.

► Example

What is the x -coordinate of the vertex of the parabola with the equation $y = (x - 6)(x - 2)$?

✓ Solution

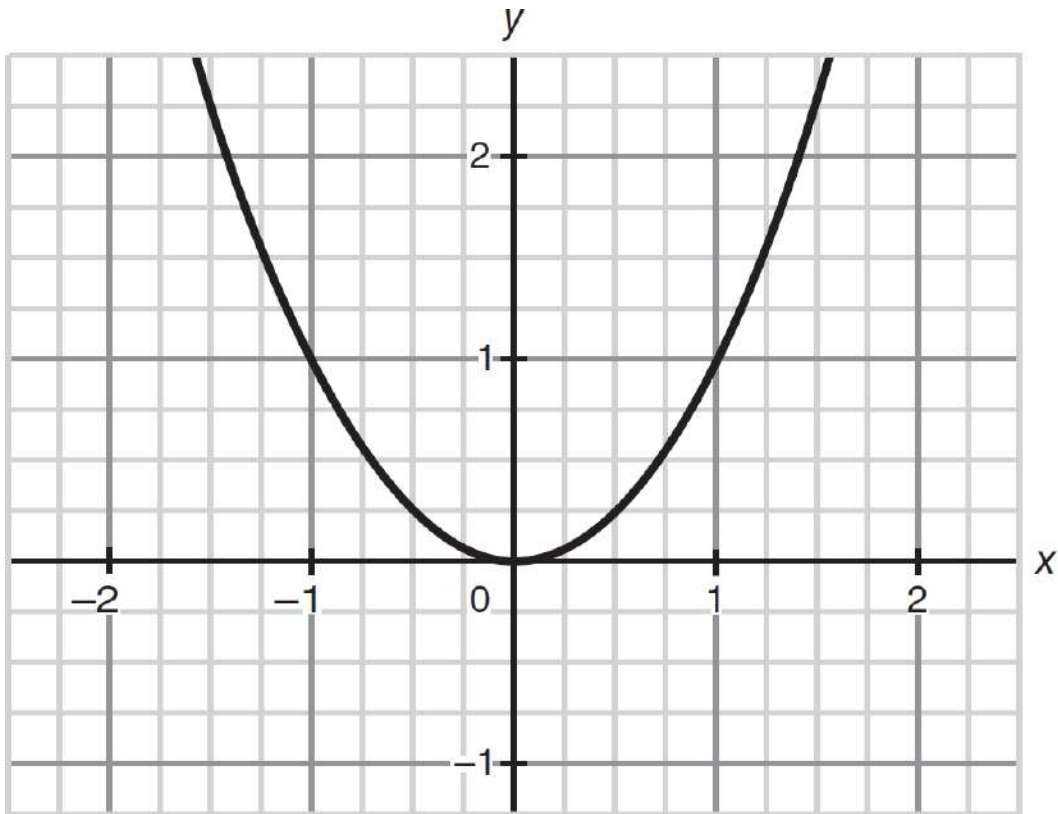
The zeros of the parabola are at x -coordinates of 6 and 2. The x -coordinate of the vertex comes halfway between these two zeros. So the x -coordinate of the vertex will be at 4 since it is halfway between 6 and 2. Here is a graph of the parabola to more easily visualize this:



Method 4: Identify the vertex from the graph.

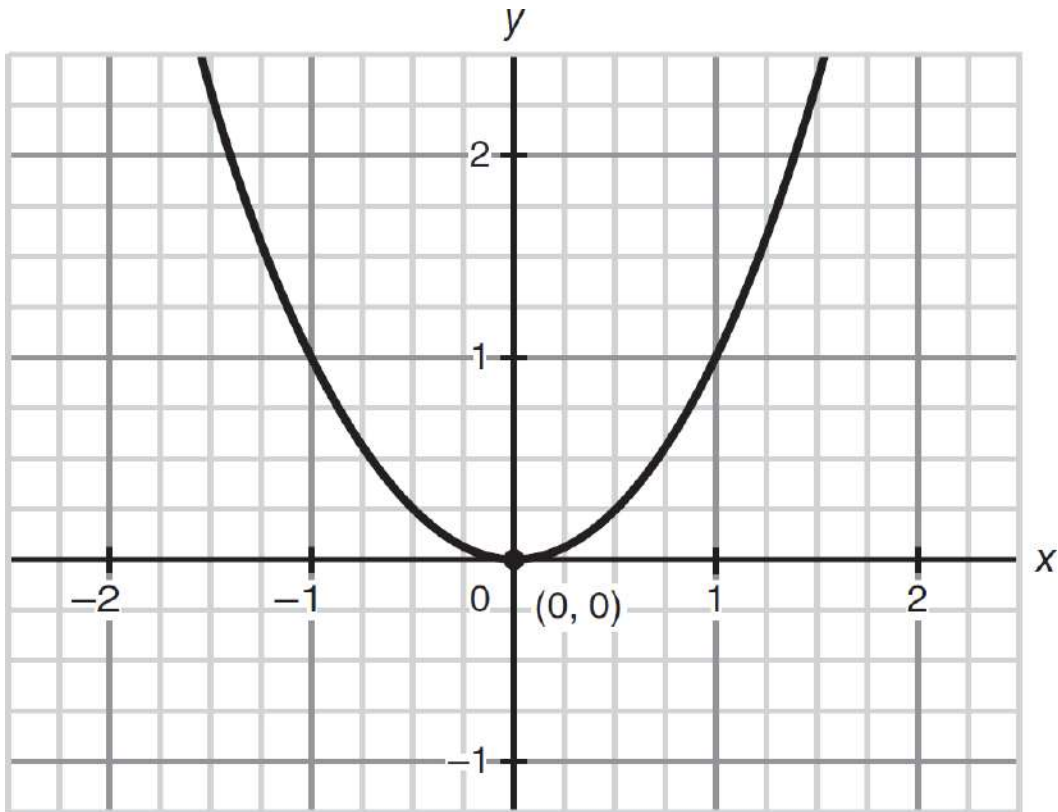
► Example

What is the vertex of the parabola with the graph below?



✓ Solution

Look for the very bottom point of the parabola to identify the vertex. The parabola has a bottom at $(0, 0)$, so the vertex of the parabola is at $(0, 0)$:



Practice

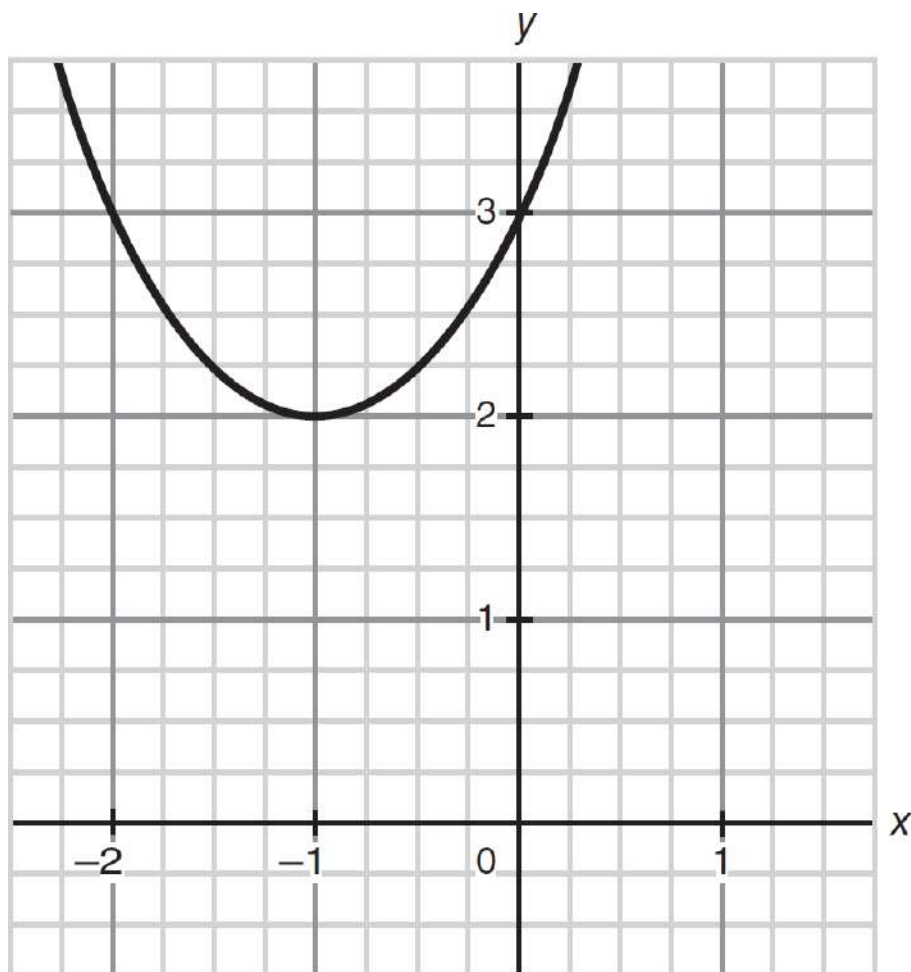
Does the parabola $y = 3(x - 2)^2 + 9$ open upwards or downwards?

What is the vertex of the parabola with the equation $y = 4(x - 1)^2 + 6$?

What is the x -coordinate of the vertex of a parabola with the equation $y = (x + 1)(x - 8)$?

What is the x -coordinate of the vertex of a parabola with the equation $y = 3x^2 - 8x + 4$?

What is the vertex of the parabola graphed below?



Solutions

The parabola is already in the form $y = a(x - h)^2 + k$, with 3 equalling a . Since a is positive, the parabola will open upwards.

The parabola is in the form $y = a(x - h)^2 + k$. From this, we can identify the values of the constants h and k to identify the vertex.

$$\begin{aligned} y &= a(x - h)^2 + k \rightarrow \\ y &= 4(x - 1)^2 + 6 \rightarrow ? \\ (h, k) &= (1, 6) \end{aligned}$$

Since the equation is already in factored form, we can identify the zeros of $y = (x + 4)(x - 8)$ as -4 and 8 . The x -coordinate of the vertex will be the midpoint of these two values:

$$\frac{-4 + 8}{2} = \frac{4}{2} = 2$$

Use the formula $x = -\frac{b}{2a}$ to solve:

$$y = ax^2 + bx + c \rightarrow$$

$$y = 3x^2 - 8x + 4 \rightarrow$$

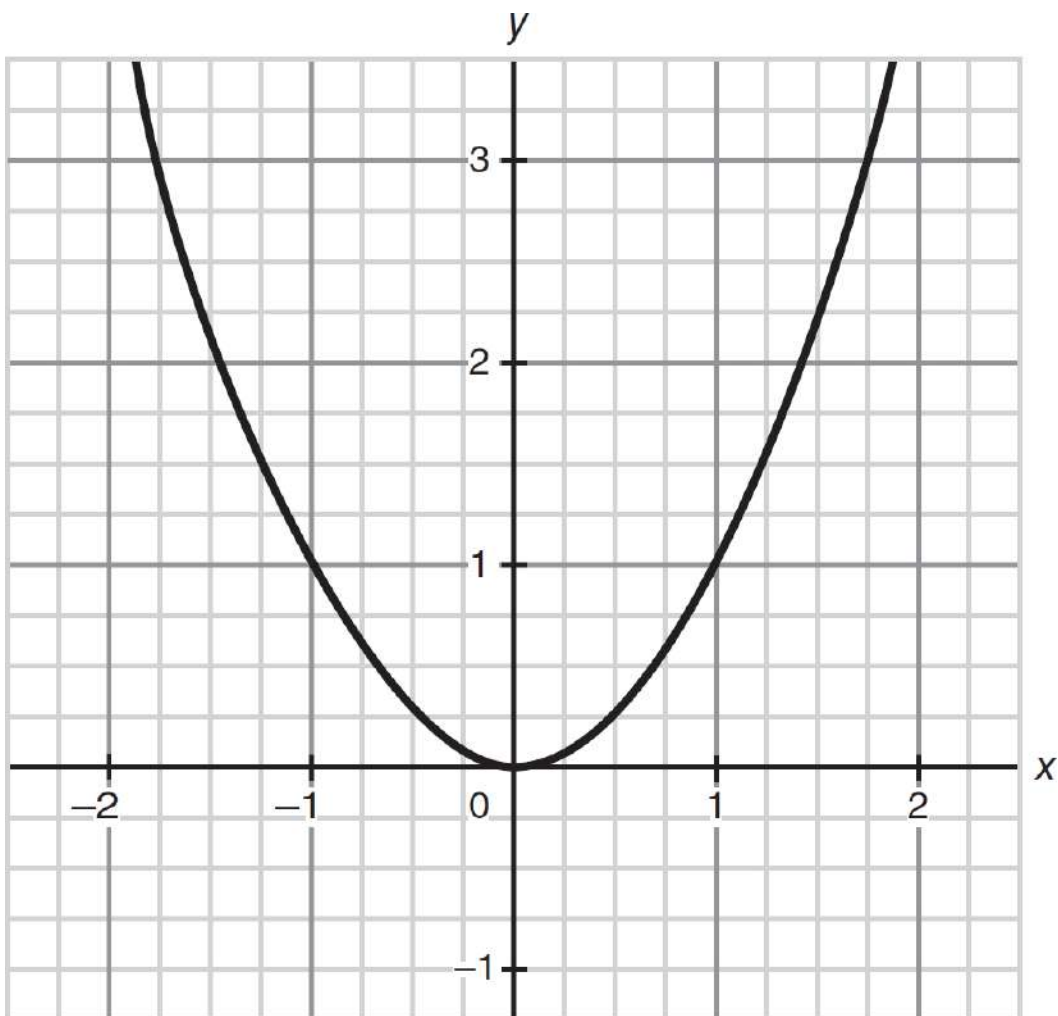
$$x = -\frac{b}{2a} \rightarrow \frac{-8}{2(3)} = \frac{8}{6} = \frac{4}{3}$$

From the graph, we can identify that the vertex of the parabola is at its lowest point, $(-1, 2)$.

Function Transformations

It is useful on some SAT Math problems to know how changes to a function's equation can impact its graph. Here are some key relationships to know.

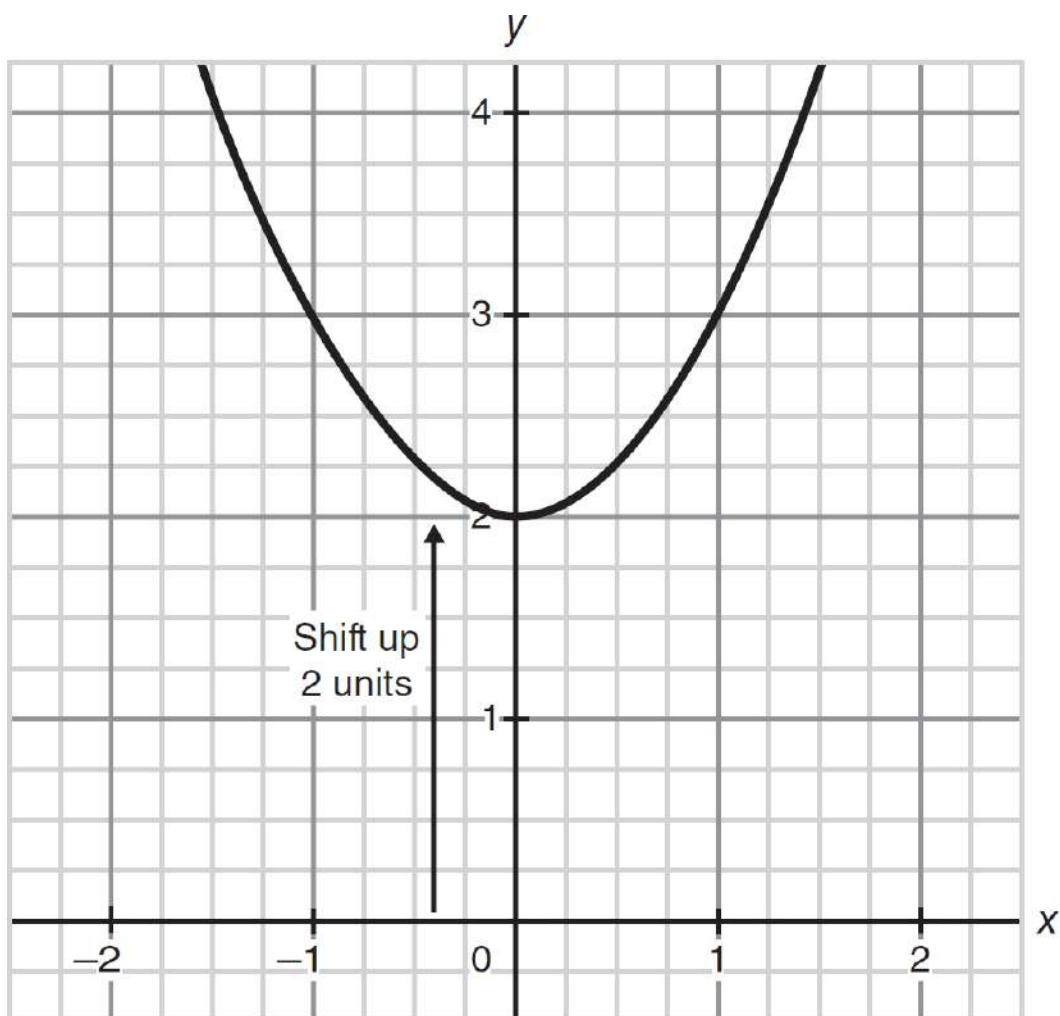
Suppose we have the function $y = x^2$ with the graph below:



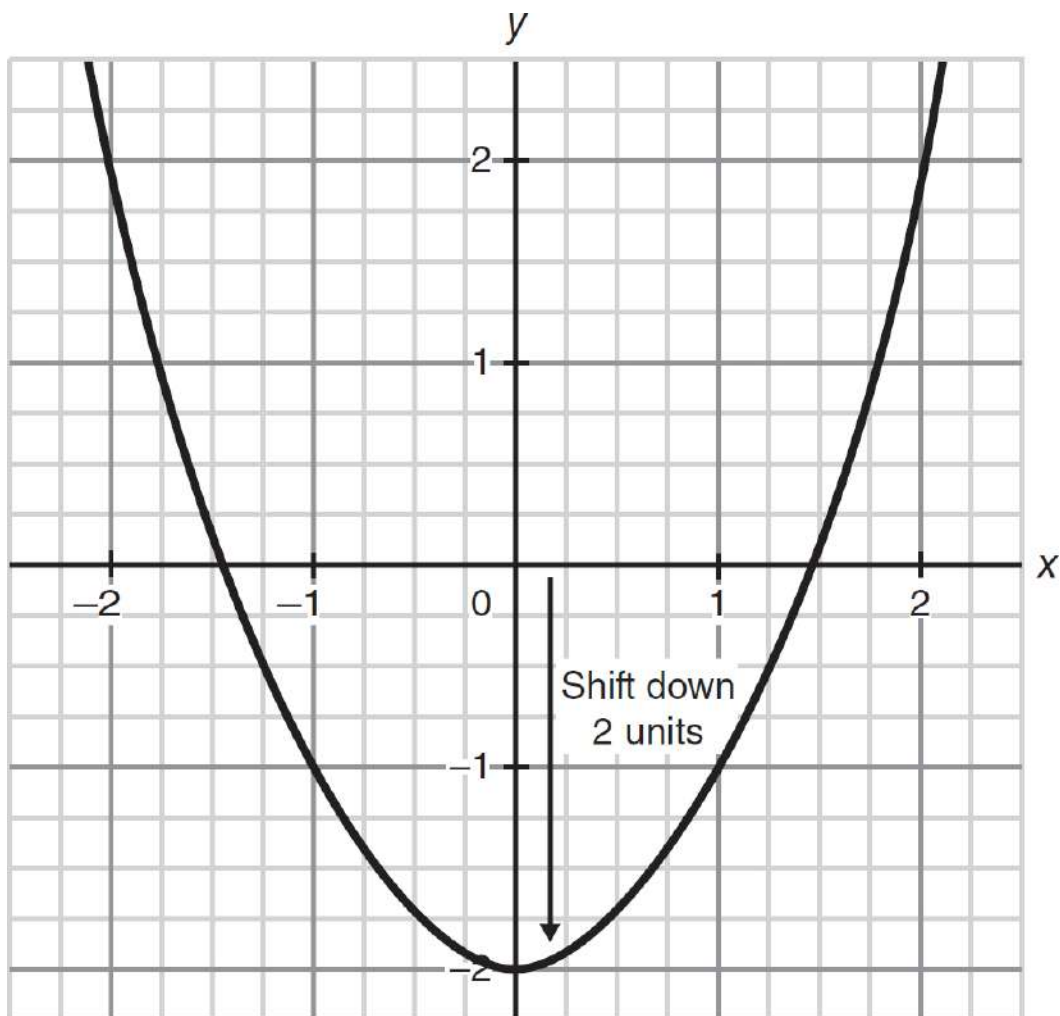
Treat h and k as constants for these rules.

Rule 1: If you transform $f(x)$ to $f(x) + k$, the function will shift UP k units.

The graph of $y = x^2 + 2$ is this:

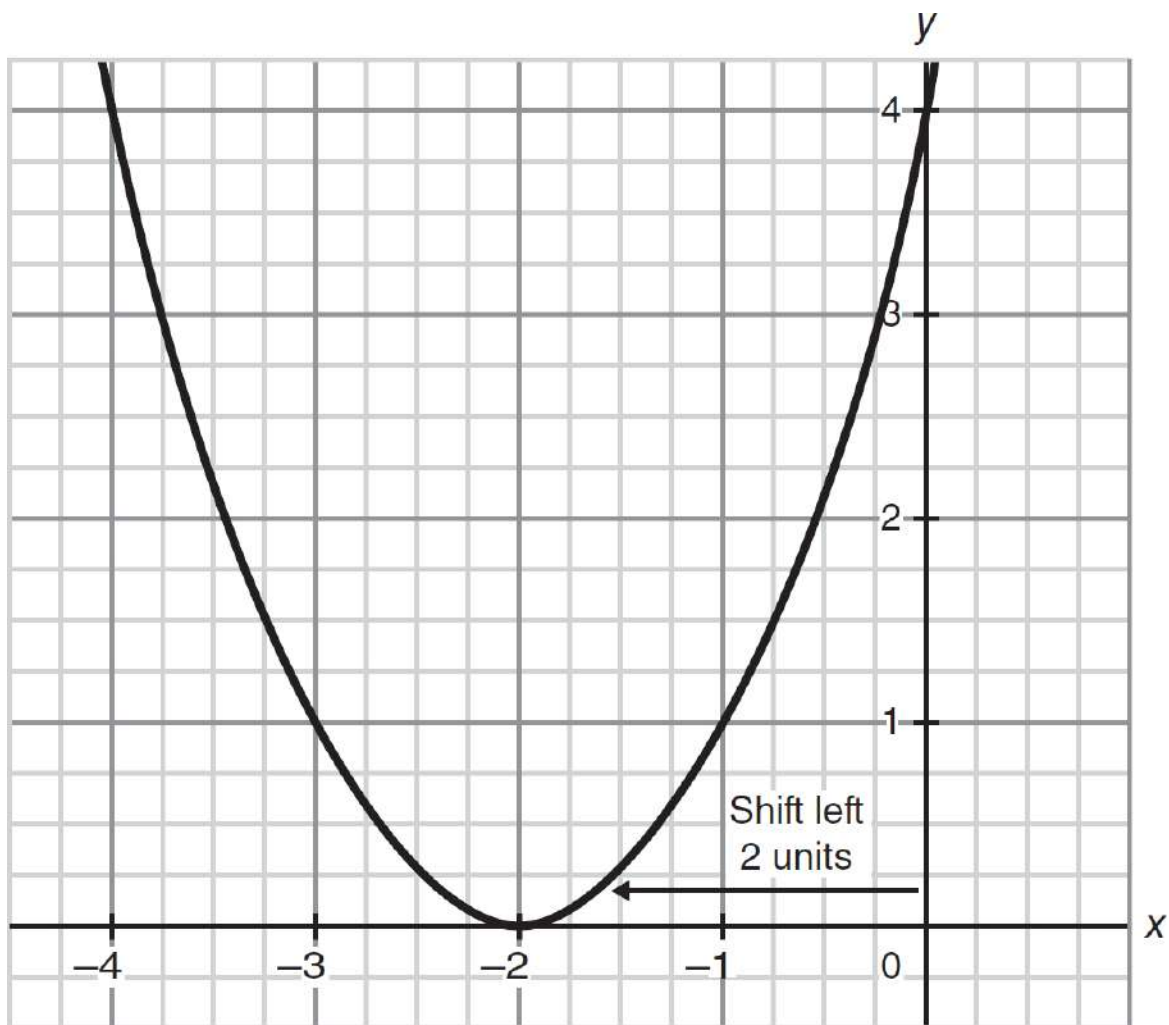


Rule 2: If you transform $f(x)$ to $f(x) - k$, the function will shift **DOWN** k units. The graph of $y = x^2 - 2$ is this:



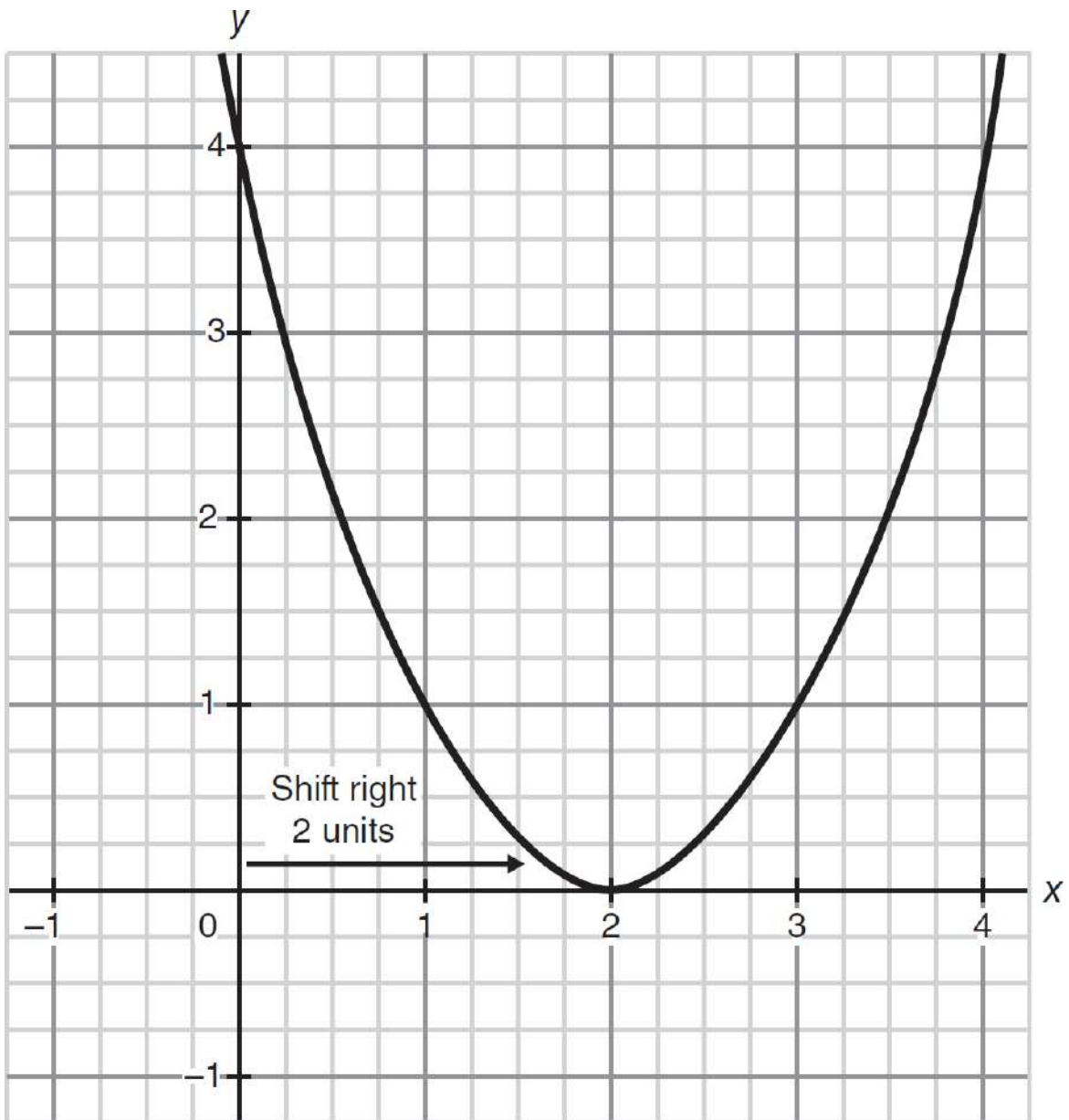
Rule 3: If you transform $f(x)$ to $f(x + h)$, the function will shift **LEFT** h units.

The graph of $y = (x + 2)^2$ is this:



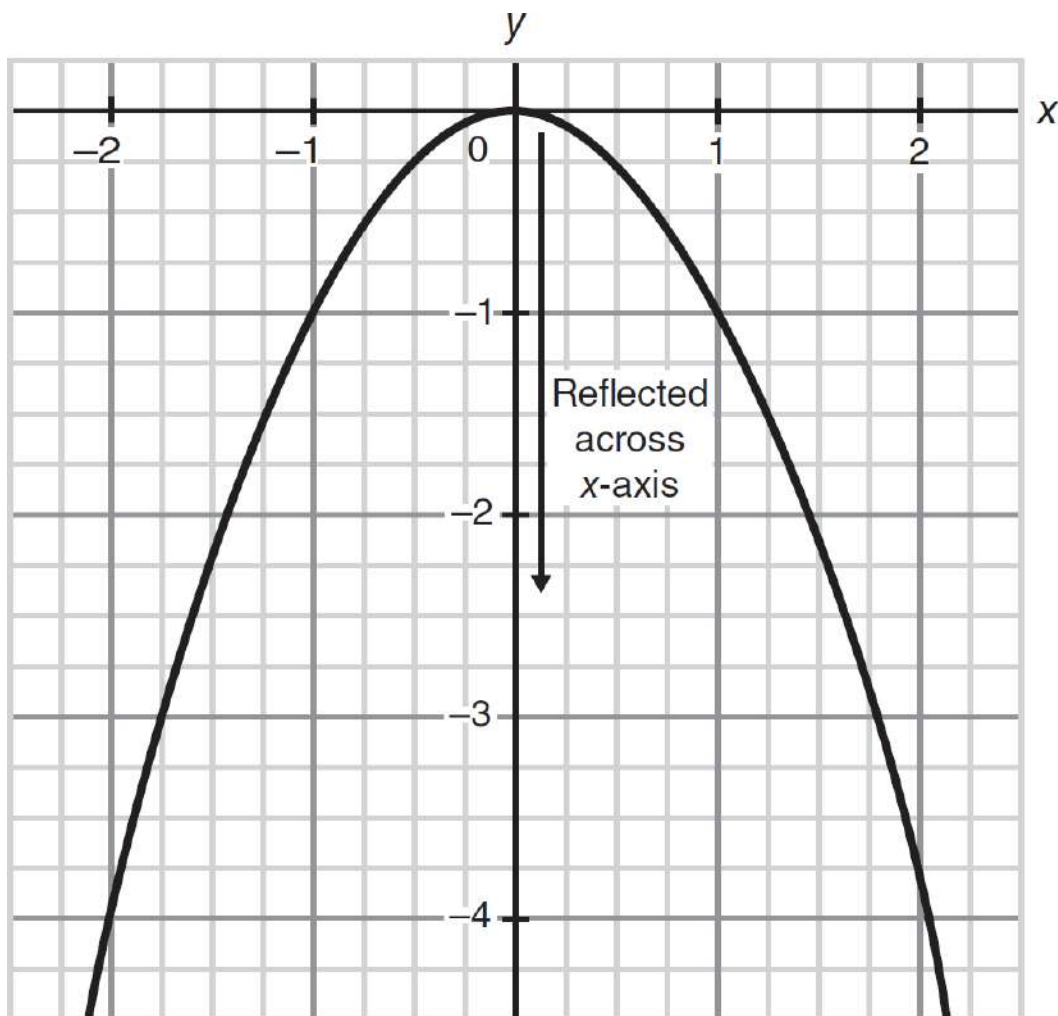
Rule 4: If you transform $f(x)$ to $f(x - h)$, the function will shift **RIGHT** h units.

The graph of $y = (x - 2)^2$ is this:



Rule 5: If you transform $f(x)$ to $-f(x)$, the function will be **REFLECTED** across the x -axis.

The graph of $y = -x^2$ is this:



Practice

f $y = 3x^3$, how will the graph of $y = 3x^3 - 5$ compare?

f $y = 4x^2$, how will the graph of $y = 4(x + 2)^2$ compare?

f $y = -2x^3$, how will the graph of $y = 2x^3$ compare?

f $y = 6x^2 - 5$, how will the graph of $y = 6(x - 1)^2 - 8$ compare?

f $y = 2(x - 4)^5 + 5$, how will the graph of $y = 2(x + 1)^5 - 5$ compare?

Solutions

t will be shifted 5 units down.

t will be shifted 2 units to the left.

t will be reflected across the x-axis.

t will be 1 unit to the right and 3 units down.
t will be 5 units to the left and 10 units down.

Visualizing Polynomial Graphs

Most of the functions you will encounter on the SAT Math are lines or parabolas. Occasionally, there will be questions involving more advanced concepts about the graphs of polynomial functions. Here are some key things to know.

Consider a polynomial with the term with the highest power term ax^n .

- The greatest possible number of turns (changes in y direction) of this function will be at most $n - 1$.
- If the constant a is positive, the function will ultimately point upwards as the x values increase; if the constant a is negative, the function will ultimately point downwards as the x values increase.
- If the exponent n is even, the ends of the function go the same direction; if the exponent n is odd, the ends of the function go in different directions.

➤ Example 1

Consider the polynomial $y = x^3 - 7x^2 + 7x + 15$.

- How many times will the polynomial turn with respect to its y values?
- Will the function ultimately point upwards or downwards?
- Will the ends of the function go in the same or different directions?

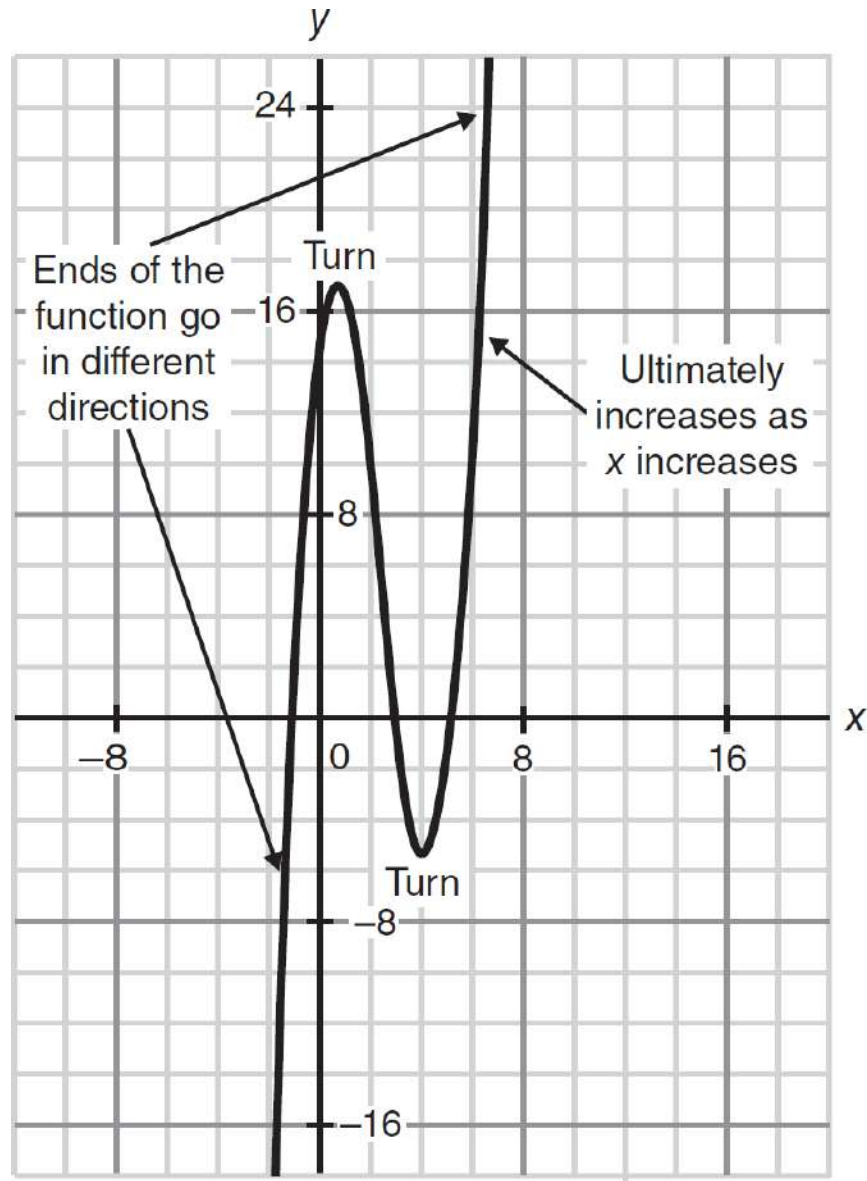
✓ Solutions

The greatest number exponential value in the polynomial is x^3 ; so the greatest possible number of turns it can have will be $3 - 1 = 2$.

Since the coefficient of x^3 is positive 1, the function will ultimately increase as the x values increase.

Since the greatest exponent in the function, 3, is odd, the ends of the function go in different directions.

All of this is easiest to see in the graph below:



➤ Example 2

Consider the polynomial $y = -x^4 + 5x^3 - 5x^2 - 5x + 6$.

- How many times will the polynomial turn with respect to its y values?

- Will the function ultimately point upwards or downwards?
- Will the ends of the function go in the same or different directions?

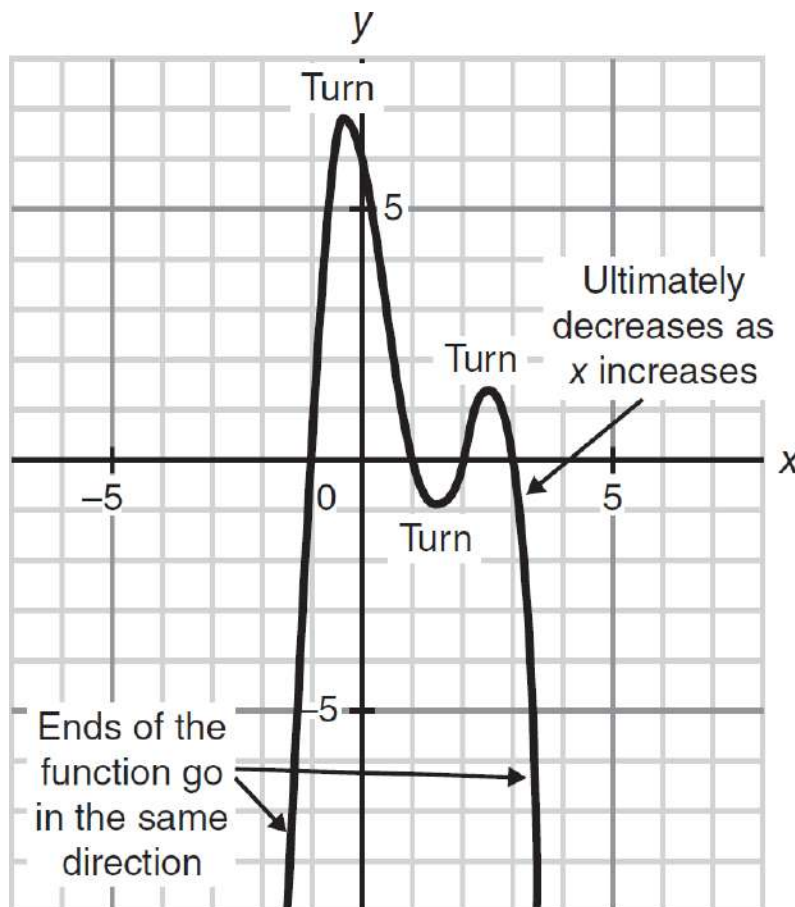
✓ Solution

The greatest number exponential value in the polynomial is x^4 , so the greatest number of turns it can have will be $4 - 1 = 3$.

Since the coefficient of x^4 is negative 1, the function will ultimately decrease as the x values increase.

Since the greatest exponent in the function, 4, is even, the ends of the function go in the same direction.

All of this is easiest to see in the graph below:



Practice

What is the most number of turns a function of the form $f(x) = a x^2$ can have?

Will the end values of the function $f(x) = x^3$ go in the same or different directions?

Will the y values in $y = 4x^5$ ultimately point upwards or downwards as x increases?

Will the end values of the function $f(x) = x^6 + 3x^5 - 2x$ go in the same or different directions?

Will the y values in $y = -6x^5 + 3x^4 + 7x^2 - 9$ ultimately point upwards or downwards as x increases?

Solutions

The greatest number exponential value in the polynomial is x^2 ; so the greatest number of turns it can have will be $2 - 1 = 1$.

Since the greatest exponent in the function, 3, is odd, the ends of the function go in different directions.

Since the coefficient of x^5 is positive 4, the function will ultimately increase as the x values increase.

Since the greatest exponent in the function, 6, is even, the ends of the function go in the same direction.

Since the coefficient of x^5 is negative 6, the function will ultimately decrease as the x values increase.

Many SAT Math problems involving zeros and parabolas will require you to use different concepts in unique ways. Write out what is given and what concepts you know to see how to put it all together.

SAT Math Strategy

Integrate your knowledge about zeros, parabolas, and polynomial graphing to solve problems.

► Example 1

$$x^2 + nx + 25 = 0$$

In the quadratic equation above, n is a constant. For what value of n will the function intersect the x -axis only once?

- (A) -13
- (B) -12
- (C) -11
- (D) -10

Solution

This question does not ask us to do something straightforward like identifying the zeros or the vertex. We need to find out what value of the constant n will make the function have only one point of intersection with the x -axis.

To have only one point of intersection with the x -axis, the function should have *only one zero*. So, if we could factor the equation and it would have only one zero in factored form, that would tell us what the constant would need to be. Notice that 25 is a perfect square—the square root of it is 5. So, if we were to write the equation in factored form, it would look like this:

$$(x - 5)(x - 5) = 0$$

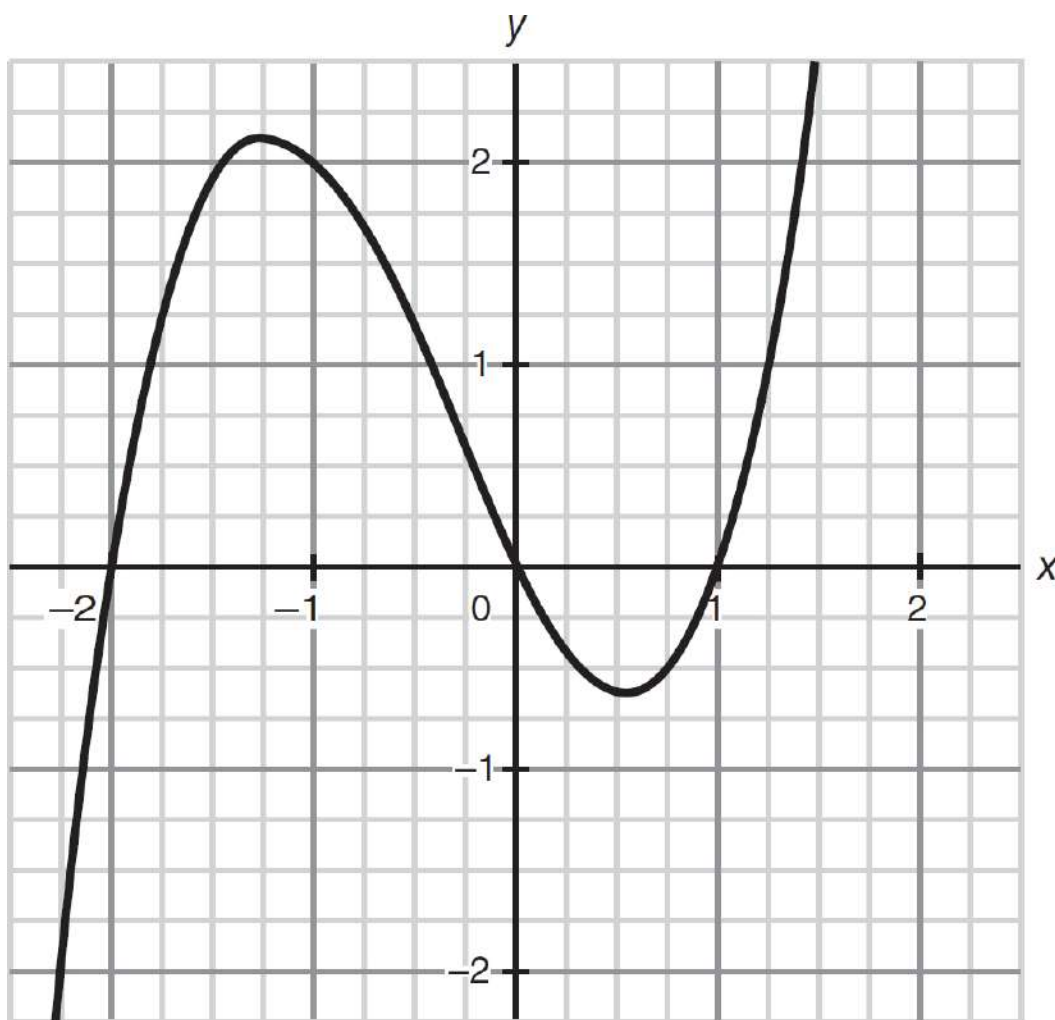
When the equation is in this form, it has only ONE zero—the number 5. Next, to identify the constant n , we can FOIL the equation to see what it looks like in the form described by the problem.

$$(x - 5)(x - 5) = 0$$

$$x^2 - 10x + 25 = 0$$

Based on this, n is -10 , choice **(D)**. So, we were able to integrate some of the different concepts we know about zeros to solve this problem.

Example 2



The graph above is represented by which of the following equations?

- (A) $y = x(x + 2)(x - 1)$
- (B) $y = x^2(x + 1)(x - 2)$
- (C) $y = x(x - 2)(x + 1)$
- (D) $y = x^2(x + 2)(x - 1)$

✓ Solution

We can integrate our knowledge about zeros and polynomial graphing to solve this problem. First, the ends of the function go in *different* directions, so the largest exponential value in the function must be *odd*. Based on this, we can eliminate choices (B) and (D) since when they are written in

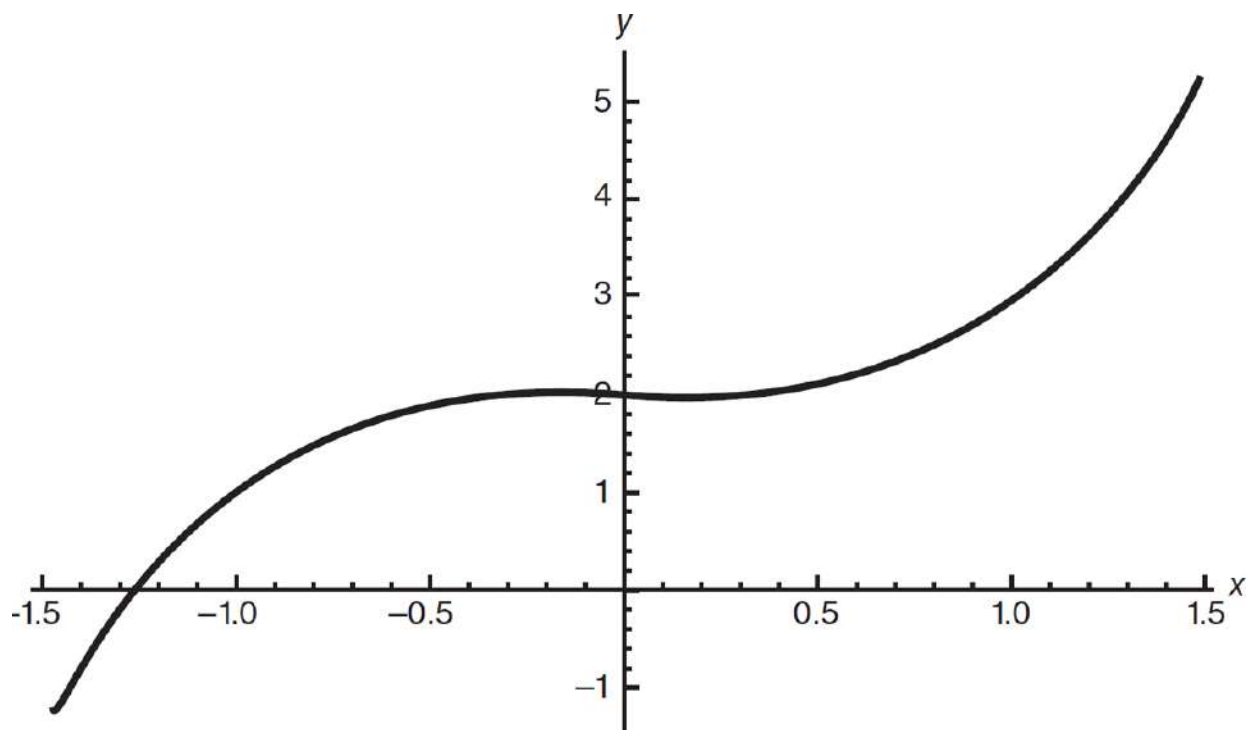
expanded form, the largest exponential value would be x^4 . Choices (A) and (C) could potentially work since when they are written in expanded form, the largest exponential value would be x^3 .

Now that we know it must be either choice (A) or (C), we can identify the zeros of the function from the graph: they are -2 , 0 , and 1 . This makes choice **(A)** correct with respect to zeros of the function, since the outside term x would work with 0 , the term $(x + 2)$ would work with -2 , and the term $(x - 1)$ would work with 1 . Therefore, the correct answer is **(A)**.

Practice Drill—Zeros, Parabolas, and Polynomial Graphing

(If timing, take about 12 minutes to complete.)

How many zeros does the function $y = x^3 + 2$, portrayed in the graph below, have?



- A) 0
- B) 1
- C) 2

D) 3

$$f(x) = (x + 4)(x - 6)$$

In the function above, what is the straight-line distance in units in the xy -coordinate plane between the function's zeros?

A) 4

B) 6

C) 10

D) 24

What is the y -intercept of the function $y = x^2 - 4$?

A) -4

B) -2

C) 1

D) 4

What is the vertex in the parabola with the equation $\frac{y + 4}{2} = (x - 5)^2$?

A) $(5, 4)$

B) $(-5, -4)$

C) $(-5, 4)$

D) $(5, -4)$

What are the x -intercepts of the function $f(x) = x^2 - 3x - 10$?

A) -2 and 5

B) -2 and -5

C) 2 and 5

D) 10 and 2

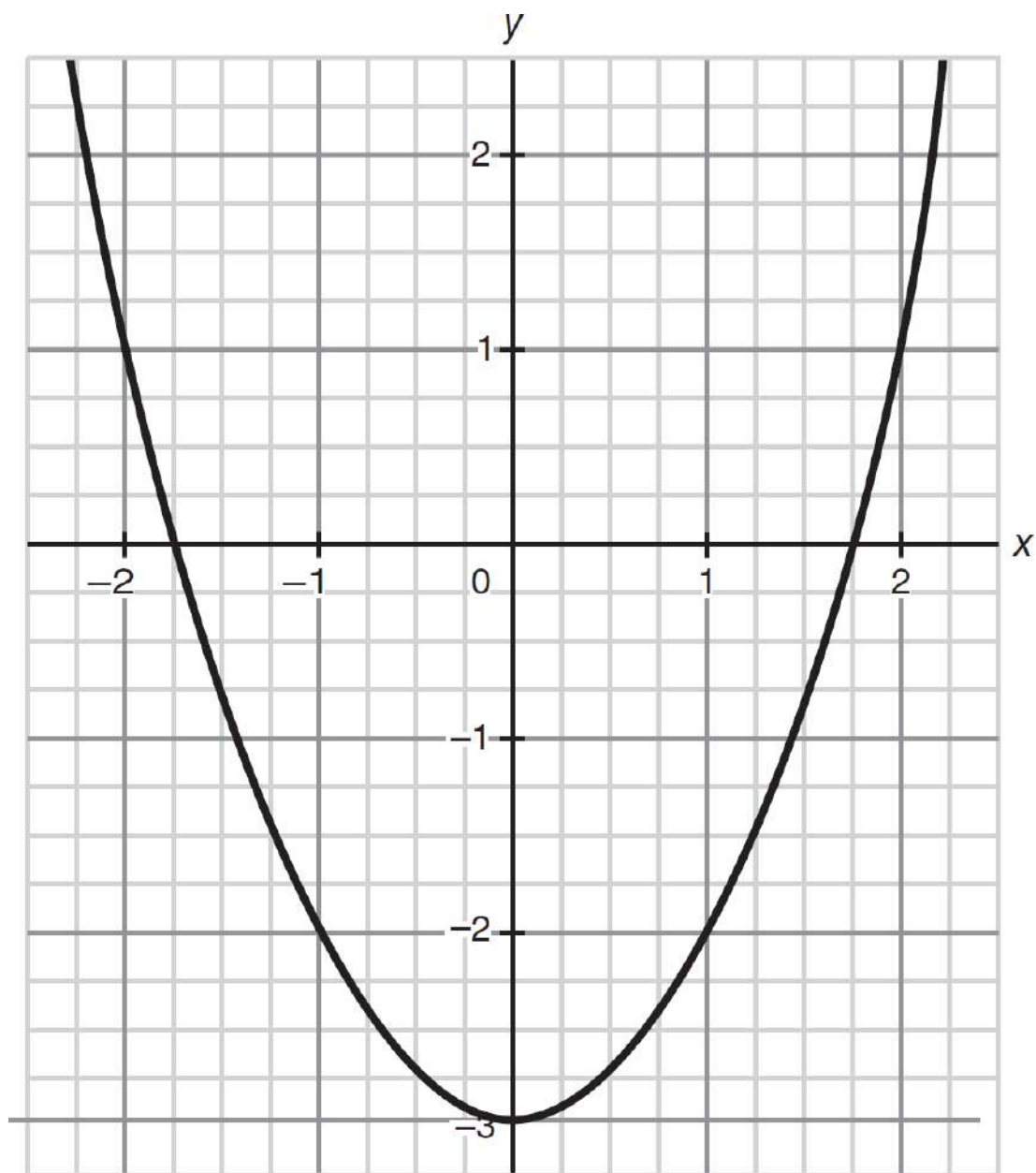
$$y = (x - 3)(x + 5)$$

The function above has a vertical axis of symmetry on which line?

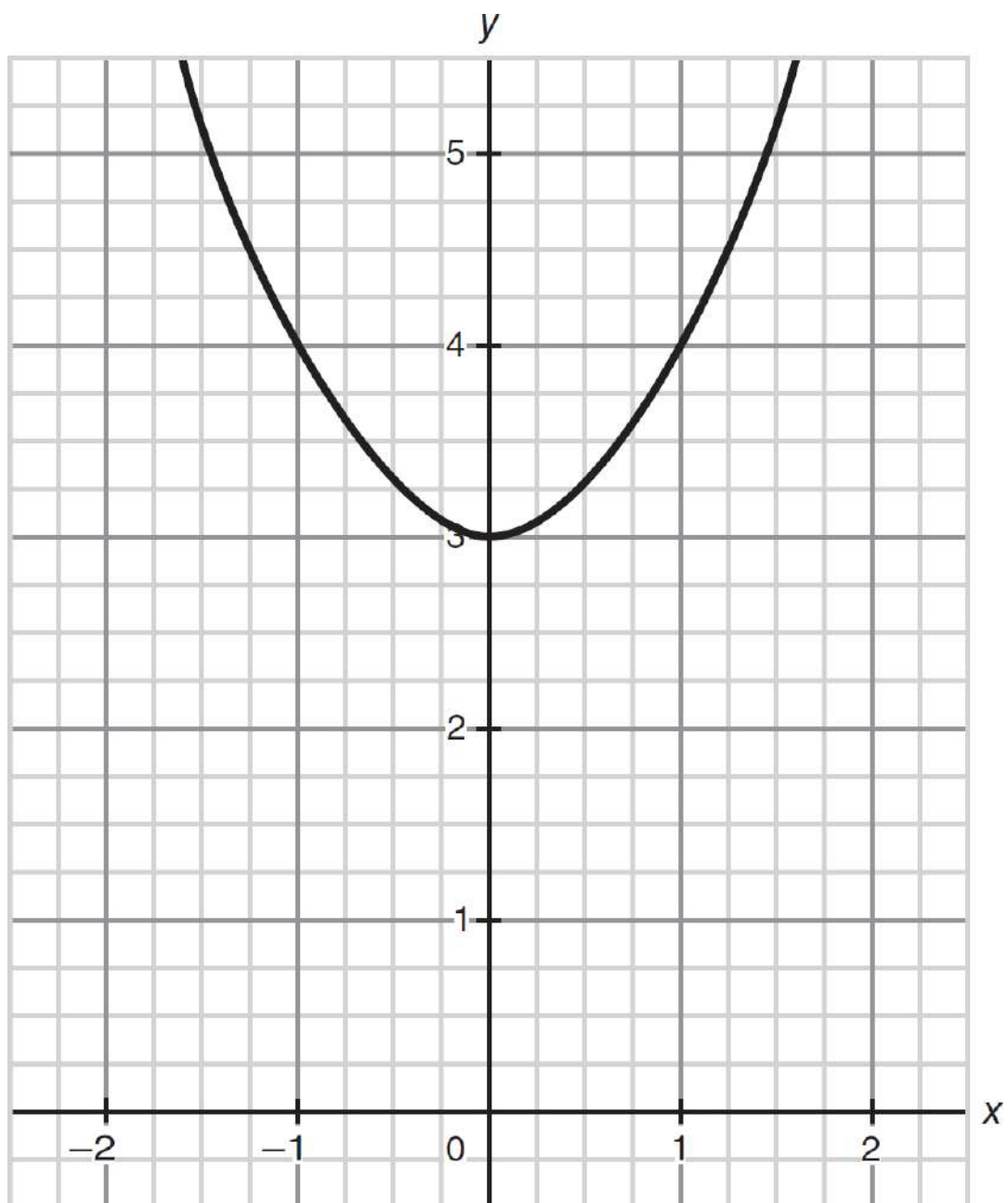
- A) $x = -5$
- B) $x = -3$
- C) $x = -1$
- D) $x = 15$

Which of the following is a graph of the equation $y = x^2 - 3$?

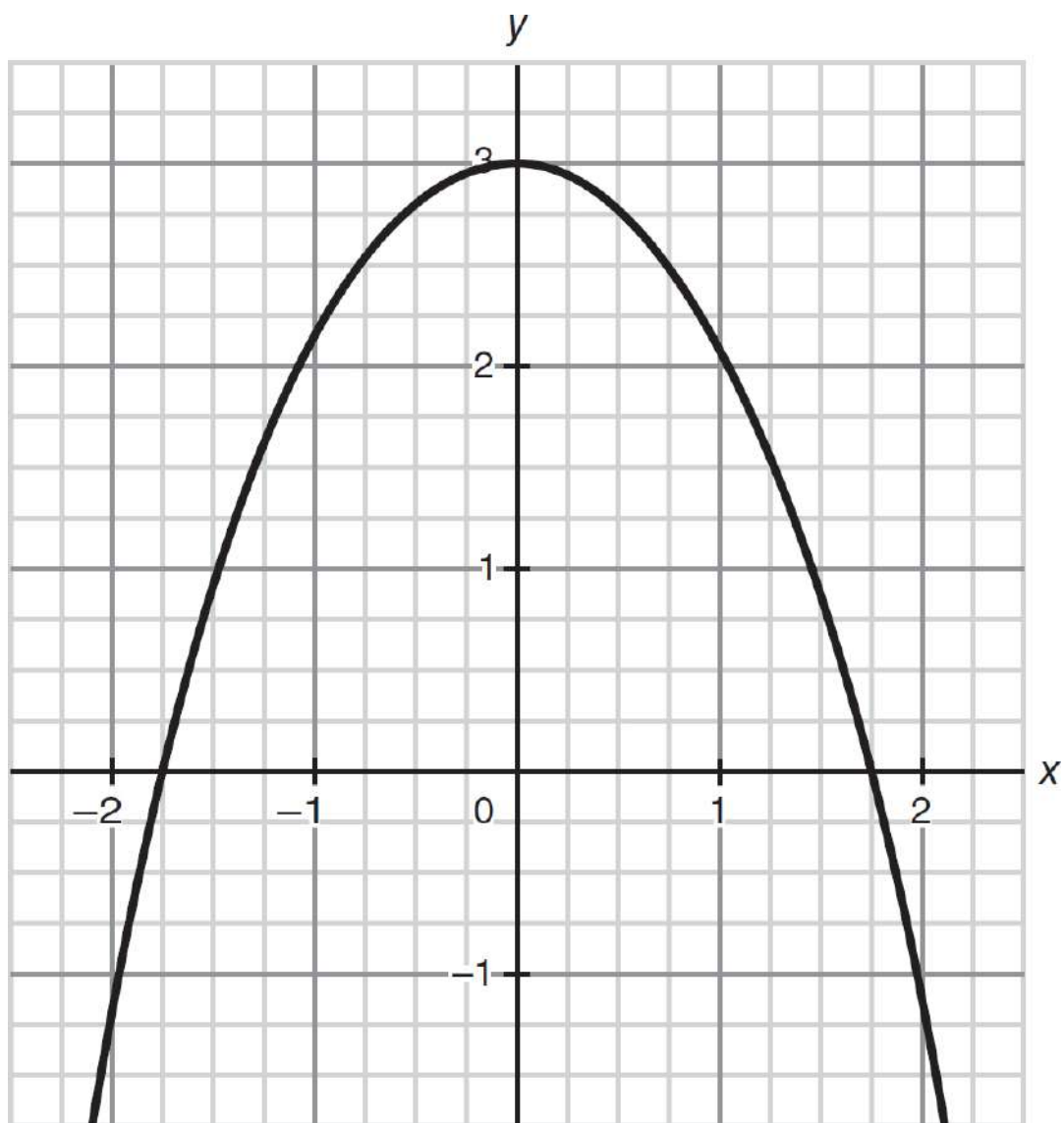
- A)



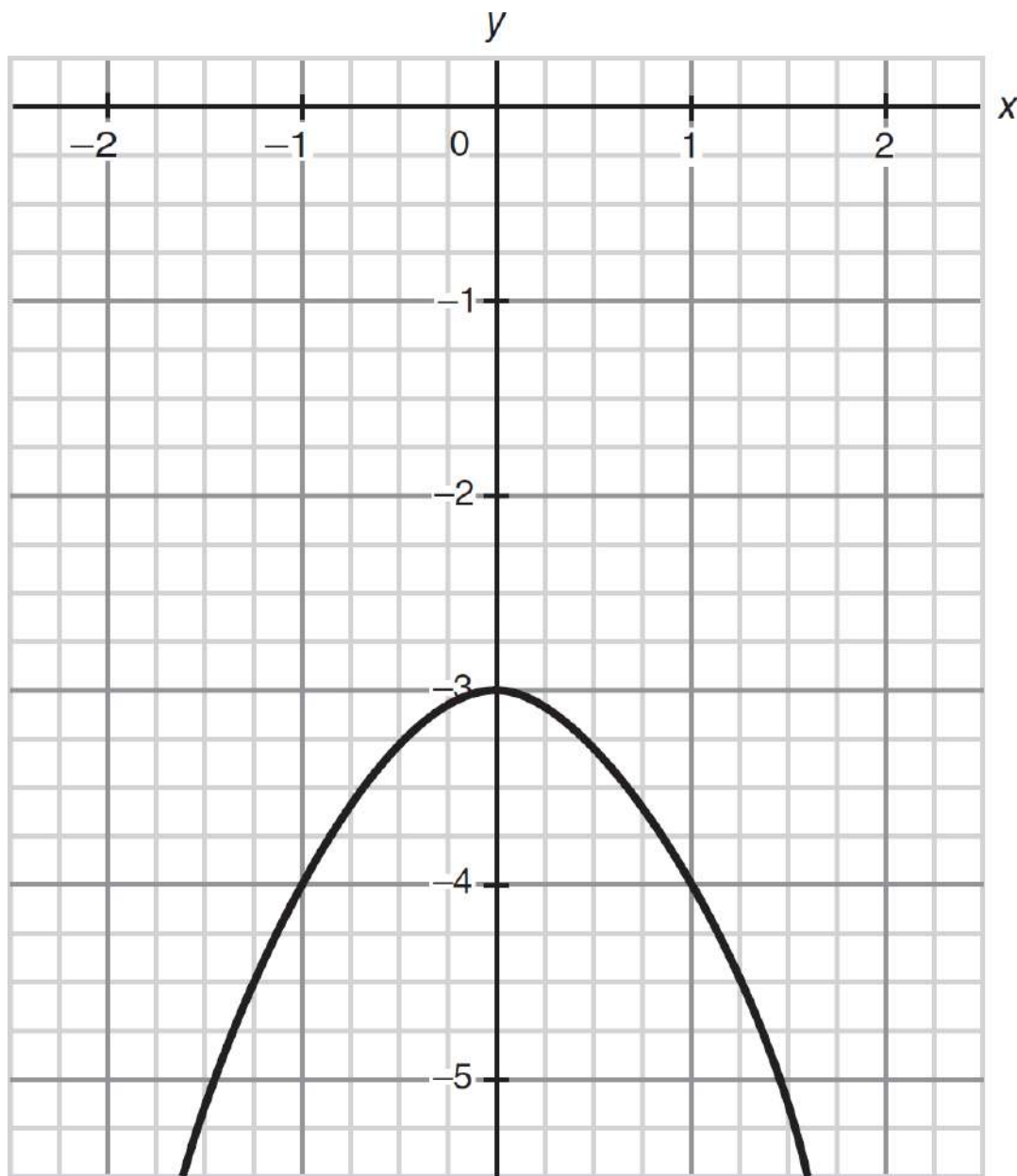
B)



C)



D)



Which of the following functions intersects the x -axis at exactly 3 points?

- A) $y = x^2 - 3$
- B) $y = x(x - 2)(x + 3)$
- C) $y = (x + 4)^3$
- D) $y = 5(x - 2)(x - 2)$

What is the y value of the y -intercept of the function $y = (5)^x$?

- A) 0
- B) 1
- C) 2
- D) 3

$f(x) = (x - 5)^2 + 4$ and $g(x)$ is a line with a slope of zero that intersects $f(x)$ at only one point, what is the point at which the two functions intersect?

- A) $(-3, -4)$
- B) $(-2, 5)$
- C) $(0, 8)$
- D) $(5, 4)$

Solutions

1. **(B)** From the graph, you can see that the function only intersects the x -axis at one point. Therefore, there is just one zero of the function.
2. **(C)** The function is already in factored form, so we can see that the zeros are at -4 and 6 . So, to find the distance between the two zeros, just measure the difference between -4 and 6 on the x -axis. This will be:

$$6 - (-4) = 10$$

3. **(A)** To find the y -intercept, we must find the y value when x is equal to zero. So, plug zero in for x to solve:

$$y = x^2 - 4 \rightarrow$$

$$y = 0^2 - 4 \rightarrow$$

$$y = -4$$

4. **(D)** Manipulate the equation to get it in the form $y = a(x - h)^2 + k$ so the vertex can be easily identified:

$$\begin{aligned}\frac{y+4}{2} &= (x-5)^2 \rightarrow \\ y+4 &= 2(x-5)^2 \rightarrow \\ y &= 2(x-5)^2 - 4\end{aligned}$$

Since the vertex corresponds to (h, k) , the vertex will be $(5, -4)$.

5. **(A)** Factor the function to identify the zeros—these are the same as the x -intercepts of the function since they are the points at which the y value is zero.

$$\begin{aligned}f(x) &= x^2 - 3x - 10 \rightarrow \\ f(x) &= (x-5)(x+2)\end{aligned}$$

Therefore, the x -intercepts are -2 and 5 .

6. **(C)** The vertical axis of symmetry will be the line that bisects the parabola through the vertex. Find the x -coordinate of the vertex and make an equation in which every x value is equal to that point. The zeros of $y = (x-3)(x+5)$ are 3 and -5 , so the x -coordinate of the vertex will be halfway between them at -1 . So, the vertical axis of symmetry will be $x = -1$.
7. **(A)** The equation is already in the form $y = a(x-h)^2 + k$, which makes it easy to identify the vertex and the direction of the parabola. The vertex is $(0, -3)$ and a is $+1$, so the parabola will open upward. The parabola graphed in choice **(A)** has this vertex and direction.
8. **(B)** Determine which of the four functions has three zeros:

$$y = x^2 - 3 \text{ has 2 zeros at } \sqrt{3} \text{ and } -\sqrt{3}.$$

$$y = x(x-2)(x+3) \text{ has 3 zeros at } 0, 2, \text{ and } -3.$$

$$y = (x+4)^3 \text{ has 1 zero at } -4.$$

$$y = 5(x-2)(x-2) \text{ has 1 zero at } +2.$$

So, choice **(B)** is correct.

9. **(B)** The y -intercept for this function is the point at which the x value is 0. So, plug 0 in for x to see the corresponding y value.

$$y = (5)^x \rightarrow$$

$$y = 5^0 = 1$$

10. **(D)** The two functions will intersect at the vertex of the parabola—if the flat, horizontal line intersected above the vertex, it would intersect at two points. Since the function is already in the form $y = a(x - h)^2 + k$, the vertex of $f(x) = (x - 5)^2 + 4$ can be identified as $(5, 4)$. This is the point at which the functions will intersect.

Advanced Math—Function Interpretation and Manipulation

Putting Variables in Terms of Other Variables

SAT Math Strategy

When asked for the value of a variable or an expression, manipulate the equation to isolate that variable or expression.

► Example 1

The energy, E , of a planet in an elliptical orbit around the sun is given by the equation

$$E = -\frac{GmM}{2a}$$

in which G is the gravitational constant, M is the mass of the sun, m is the mass of an orbiting planet, and a is the length of the semimajor axis of the ellipse made by the planetary orbit. What is the value of a planet's mass in terms of the other values?

✓ Solution

Typically, when you solve an equation, you find the numerical value of x or some other variable. In a question like this, you should isolate the variable for mass, m , to see what m is “in terms of” the other variables.

$$E = -\frac{GmM}{2a} \rightarrow$$

Multiply both sides by $2a \rightarrow$

$$2aE = -GmM \rightarrow$$

Divide both sides by $-GM \rightarrow$

$$m = -\frac{2aE}{GM}$$

► Example 2

What is the value of $2n - 1$ in the equation below?

$$\frac{2n - 1}{3} + 11 = 8n - 4$$

✓ Solution

As a general rule, *when the SAT asks you to find the value of an expression, there will be a straightforward way to find it by treating the expression as a whole unit.* This problem is no exception. We do not need to solve for n individually—instead, we can manipulate the equation to isolate the expression $2n - 1$:

$$\frac{2n - 1}{3} + 11 = 8n - 4 \rightarrow$$

Factor the right side $\rightarrow$

$$\frac{(2n - 1)}{3} + 11 = 4(2n - 1) \rightarrow$$

Multiply both sides by 3 $\rightarrow$

$$(2n - 1) + 33 = 12(2n - 1) \rightarrow$$

Subtract $(2n - 1)$ from both sides $\rightarrow$

$$\begin{aligned} 33 &= 11(2n - 1) \rightarrow \\ 3 &= (2n - 1) \end{aligned}$$

So, the value of $2n - 1$ is 3.

Identifying Constants

SAT Math Strategy

Recognize that the SAT will make it easy to identify constant terms.

► Example 1

If $(x + 4)^2 + 5 = ax^2 + bx + c$, what is the sum of the constants a , b , and c ?

✓ Solution

You do not need to use any special formulas to solve this. Just expand the left side of the expression and simplify so you can identify the constants a , b , and c :

$$\begin{aligned} (x + 4)^2 + 5 &\rightarrow \\ (x + 4)(x + 4) + 5 &\rightarrow \\ x^2 + 4x + 4x + 16 + 5 &\rightarrow \\ x^2 + 8x + 16 + 5 &\rightarrow \\ x^2 + 8x + 21 & \end{aligned}$$

So,

$$a = 1, b = 8, \text{ and } c = 21$$

$$1 + 8 + 21 = 30$$

► Example 2

$$f(x) = (x - 4)(x + 11)$$

In the equation above, which of these values is displayed as a constant or constants?

- (A) y -intercepts
- (B) x -intercepts
- (C) x -coordinate of the vertex
- (D) y -coordinate of the vertex

✓ Solution

In the function, we see the numbers 4 and 11 displayed as constants. What do we know about this function based on what we have previously covered? The *zeros* of the function are going to be 4 and -11 . Since a zero is where the function intersects the x -axis, we can say that the x -intercepts are displayed as constants in the function.

Interpreting Functions

SAT Math Strategy

Apply what you have already learned about algebra and graphs to interpret functions—you will only be tested on what is reasonable to know.

› Example 1

A company's profit, P , for selling n products is modeled by the following function:

$$P = -10n^2 + 300n - 1,000$$

How many products should the company sell to achieve its maximum profit?

- (A) 10
- (B) 12
- (C) 15
- (D) 18

✓ Solution

While it would be possible to work your way backwards from the solutions, that could be time-consuming in this case. Instead, use your knowledge of quadratic functions to solve. The maximum of a parabola that is facing downwards can be found at the vertex of the parabola. The n^2 term is the largest power term, and it has -10 as its coefficient, meaning the parabola will face downwards. So, if we can find the vertex of the parabola, we can find the x -coordinate at which the parabola is at its maximum.

Use the formula $x = -\frac{b}{2a}$ to find the vertex since the equation is in standard form. The constant a will be -10 , and the constant b will be 300 , and we can substitute n for x .

$$\begin{aligned}x &= -\frac{b}{2a} \rightarrow \\n &= -\frac{b}{2a} \rightarrow \\n &= -\frac{300}{2(-10)} = \frac{300}{20} = 15\end{aligned}$$

Thus, the number of products that should be sold to maximize the profit is 15, choice (C).

▸ Example 2

In the equation $n x^2 + m = 5 x^2 + 4$, there are infinitely many solutions for x . What is the product of the constants m and n ?

- (A) 10
- (B) 14
- (C) 18

(D) 20

Solution

Think through what you know about equations. For there to be infinitely many solutions, the equation can be formatted so that what is on the left side and the right side are identical. For example, $x + 5 = x + 5$ would have infinitely many solutions since any number can be plugged in for x to make this equation true.

So, let's apply the same idea to this problem. For this equation to have infinitely many solutions, the two sides should be equal:

$$nx^2 + m = 5x^2 + 4 \rightarrow$$

$$5x^2 + 4 = 5x^2 + 4$$

Therefore, n equals 5 and m equals 4. The product comes from multiplying the two numbers together. Thus, the product of 5 and 4 is $5 \times 4 = 20$, choice **(D)**.

Practice Drill #1—Function Interpretation and Manipulation

(If timing, take about 12 minutes to complete.)

In the equation $E = mc^2$, what is the value of c in terms of the other values?

- A) Em
- B) $\frac{m}{E}$
- C) $\sqrt{\frac{E}{m}}$
- D) $\sqrt{\frac{m}{E^2}}$

The formula to model the height, h , in meters of an object t seconds after being dropped is $h(t) = -4.9t^2 + h_0$ in which h_0 is the height from which the object is dropped. If the height of an object 3 seconds after being dropped is 100, what is the initial height of the object?

- A) 9.8 meters
- B) 17.2 meters
- C) 24.4 meters
- D) 44.1 meters

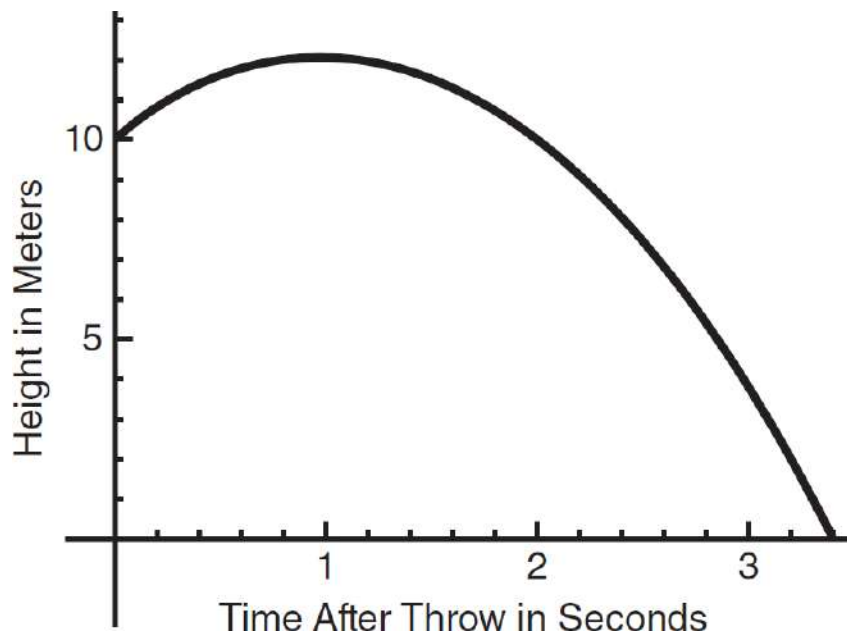
Body mass index is calculated by taking the mass of a person in pounds and the height of a person in inches and plugging these values into this formula:

$$BMI = \frac{mass}{height^2} \times 703$$

What is the mass of a person in terms of the other values?

- A) $mass = \frac{BMI \times height^2}{703}$
- B) $mass = \frac{BMI \times height}{703}$
- C) $mass = \frac{703}{BMI \times height^2}$
- D) $mass = \frac{703}{BMI \times height}$

The height of a rock thrown off a ledge relative to the ground beneath the edge is graphed below.



At what time did the rock reach its maximum height?

- A) At the time of the throw
- B) 1 second after the throw
- C) 2 seconds after the throw
- D) 3.5 seconds after the throw

If $x - 5 = \frac{2x^2 + ax - 20}{2x + 4}$, what is the value of the constant a ?

- A) -6
- B) -3
- C) $\frac{1}{2}$
- D) 8

The moon orbits Earth once approximately every 27 days. Titan, a moon of the planet Saturn, orbits Saturn once approximately every 16 days. For every full planetary orbit the moon of Earth makes, M , how many orbits of Saturn (or fractions thereof) will Titan make of Saturn, T ?

- A) $T = \frac{16}{27}M$
- B) $T = \frac{4}{9}M$
- C) $T = \frac{27}{16}M$
- D) $T = \frac{9}{4}M$

Fill-In Practice: Write your answer in the underlined blank under each question.

The number of geese at a lake tripled each year, when compared to the number of geese on January 1 of each year between January 1, 2017, and January 1, 2020. If there were 810 geese on January 1, 2020, how many geese would there have been on January 1, 2017?

Answer: _____

What is the y-intercept of the graph of the following equation?

$$y = \frac{2x^2 - 18}{x - 3}$$

Answer: _____

If $2(3x^2 - 4x + 2) = ax^2 + bx + c$, what is the value of c ?

Answer: _____

$$f = \frac{4x^2 - 36}{2x + 6}$$

What is the slope of the line formed by the equation above?

Answer: _____

Solutions

1. **(C)** Isolate the c to arrive at your solution:

$$E = mc^2 \rightarrow$$

$$\frac{E}{m} = c^2 \rightarrow$$

$$\sqrt{\frac{E}{m}} = \sqrt{c^2} \rightarrow$$

$$c = \sqrt{\frac{E}{m}}$$

2. **(D)** The value of h_0 is the initial height of the object. So, plug in 3 for t and 0 for the final height, $h(t)$:

$$h(t) = -4.9t^2 + h_0 \rightarrow$$

$$0 = -4.9(3)^2 + h_0 \rightarrow$$

$$0 = -44.1 + h_0 \rightarrow$$

$$h_0 = 44.1$$

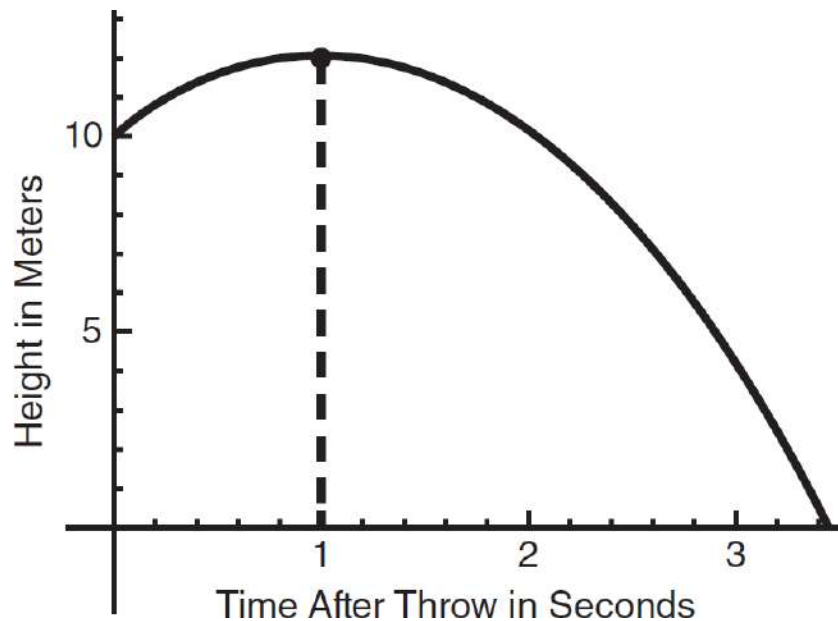
3. **(A)** Manipulate the equation until mass is isolated.

$$BMI = \frac{mass}{height^2} \times 703 \rightarrow$$

$$\frac{BMI}{703} = \frac{mass}{height^2} \rightarrow$$

$$mass = \frac{BMI \times height^2}{703}$$

4. **(B)** Find the point on the graph where the height is at its maximum:



So, the rock reached its maximum height at 1 second after the throw.

5. **(A)**

$$x - 5 = \frac{2x^2 + ax - 20}{2x + 4} \rightarrow$$

$$(x - 5)(2x + 4) = 2x^2 + ax - 20 \rightarrow$$

$$2x^2 - 10x + 4x - 20 = 2x^2 + ax - 20 \rightarrow$$

$$2x^2 - 6x - 20 = 2x^2 + ax - 20$$

The constant a corresponds to -6 .

6. **(C)** It takes the moon 27 days to make a complete orbit of Earth, and it takes Titan 16 days to make a complete orbit of Saturn. So, if the

moon has made one full orbit, 27 days have gone by—in that time, Titan would have made nearly two orbits since it orbits more quickly. To be precise, it would make $\frac{27}{16}$ orbits. Therefore, the answer is choice (C) $T = \frac{27}{16}M$.

Another way to think about this: Titan makes $\frac{1}{16}$ of an orbit in one day. Since it takes the moon 27 days to make a complete orbit, Titan would make $\frac{1}{16} \times 27 = \frac{27}{16}$ of an orbit in the time it takes the moon to make a single orbit.

7. **(30)** Since the number of geese triples each year, work your way backwards to the number of geese in 2017 by taking $\frac{1}{3}$ of each previous year's total.

In 2020, there were 810 geese.

In 2019, there would be $\frac{1}{3}$ of 810, which is 270.

In 2018, there would be $\frac{1}{3}$ of 270, which is 90.

And in 2017, there would be $\frac{1}{3}$ of 90, which is 30.

8. **(6)**

$$y = \frac{2x^2 - 18}{x - 3} \rightarrow$$

$$y = \frac{2(x + 3)(x - 3)}{x - 3} \rightarrow$$

Cancel the $x - 3 \rightarrow$

$$y = 2(x + 3) \rightarrow$$

$$y = 2x + 6$$

Since the equation is now in slope-intercept form, we can see that the y -intercept is 6. Alternatively, just plug in 0 for x and evaluate the y value.

9. (4) Distribute the 2 to put the left-hand side of the equation in a similar form to the right-hand side:

$$2(3x^2 - 4x + 2) = ax^2 + bx + c \rightarrow$$

$$6x^2 - 8x + 4 = ax^2 + bx + c$$

So, the constant c is equal to the constant 4.

10. (2) Manipulate this equation to get it in slope-intercept form:

$$y = \frac{4x^2 - 36}{2x + 6} \rightarrow$$

$$y = \frac{4(x^2 - 9)}{2(x + 3)} \rightarrow$$

$$y = 2 \frac{(x + 3)(x - 3)}{(x + 3)} \rightarrow$$

$$y = 2 \frac{\cancel{(x + 3)}(x - 3)}{\cancel{(x + 3)}} \rightarrow$$

$$y = 2x - 6$$

So, the slope, m , of the line is 2 since the equation is in the form $y = mx + b$.

Practice Drill #2—Function Interpretation and Manipulation

(If timing, take about 12 minutes to complete.)

In physics, force, F , is equal to the mass, m , multiplied by acceleration, a :

$$F = ma$$

What is acceleration in terms of force and mass?

A) $a = F + m$

B) $a = \frac{m}{F}$

C) $a = \frac{1}{Fm}$

D) $a = \frac{F}{m}$

In a sphere, the surface area, S , is calculated using the formula $S = 4\pi r^2$, where r is the radius of the sphere. What is the value of the radius in terms of the surface area?

- A) $r = \frac{S}{4\pi}$
- B) $r = 16\pi S^2$
- C) $r = \sqrt{\frac{S}{4\pi}}$
- D) $r = \sqrt{\frac{4\pi}{S}}$

If a is a positive constant, what is the value of a in the equation below?

$$(2x + a)(3x + a) = 6x^2 + 5ax + 16$$

- A) 4
- B) 5
- C) 6
- D) 30

The solute potential of a solution Ψ_s is calculated as follows:

$$\Psi_s = -iCRT$$

In this equation, i is the ionization constant, C is the molar concentration, R is the pressure constant, and T is the temperature in degrees Kelvin. What is the molar concentration in terms of the other values?

- A) $C = -\frac{R\Psi_s}{iT}$
- B) $C = -\frac{TR}{i\Psi_s}$
- C) $C = -\frac{iRT}{\Psi_s}$
- D) $C = -\frac{\Psi_s}{iRT}$

The height, h , of a ball thrown from the top of a ladder that is 10 feet above the ground, and t seconds after being thrown, is calculated using this function:

$$h(t) = -16t^2 + 20t + 10$$

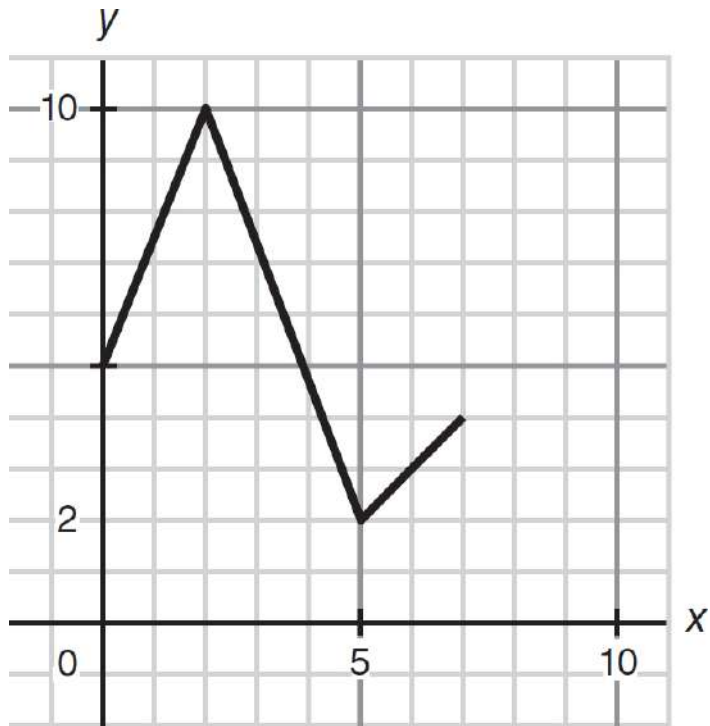
After how many seconds would the ball reach its maximum height?

- A) 0.25 seconds
- B) 0.48 seconds
- C) 0.625 seconds
- D) 0.895 seconds

$$(2x - 3y)(4x + y) = 8x^2 + axy + bxy - 3y^2$$

In the equation with variables x and y above, what is the sum of the constants a and b ?

- A) -10
- B) -6
- C) 4
- D) 12



The graph above represents all the values for the function $g(x)$ as graphed in the xy -coordinate plane. What is the difference between the minimum and the maximum value of the function?

- A) 5
- B) 7
- C) 8
- D) 10

$$k(a x^2 + b x + 5) = 6 x^2 - 21 x + 15$$

In the expression above, in which k , a , and b are constants, what is the value of k ?

- A) 3
- B) 7
- C) 21
- D) 75

$f(x) = x^2 - 4$ and $f(x + n) = x^2 + 6x + 5$, what is the value of the constant n ?

- A) 1
- B) 2
- C) 3
- D) 4

$2y = 3x - 4$ and $y + 1 = z - 5$, what is x in terms of z ?

- A) $x = \frac{2}{3}z - \frac{8}{3}$
- B) $x = \frac{4}{5}z + 2$
- C) $x = -\frac{3}{5}z - \frac{4}{9}$
- D) $x = -3z + \frac{1}{4}$

Solutions

1. **(D)** Manipulate the equation so that a is isolated:

$$F = ma \rightarrow$$

Divide both sides by $m \rightarrow$

$$a = \frac{F}{m}$$

2. **(C)** Solve the equation for the radius, r : In a sphere, the surface area, S , is calculated using the formula $S = 4\pi r^2$, where r is the radius of the sphere. What is the value of the radius in terms of the surface area?

$$S = 4\pi r^2 \rightarrow$$

$$r^2 = \frac{S}{4\pi} \rightarrow$$

$$\sqrt{r^2} = \sqrt{\frac{S}{4\pi}} \rightarrow$$

$$r = \sqrt{\frac{S}{4\pi}}$$

3. **(A)** FOIL the left-hand side of the equation so you can identify what values are equivalent:

$$\begin{aligned}(2x + a)(3x + a) &= 6x^2 + 5ax + 16 \rightarrow \\ 6x^2 + 2ax + 3ax + a^2 &= 6x^2 + 5ax + 16 \rightarrow \\ 6x^2 + 5ax + a^2 &= 6x^2 + 5ax + 16 \rightarrow \\ a^2 &= 16\end{aligned}$$

So, a could equal 4. Plugging this back in, you can see that having 4 for a makes the equation work:

$$\begin{aligned}(2x + a)(3x + a) &= 6x^2 + 5ax + 16 \rightarrow \\ (2x + 4)(3x + 4) &= 6x^2 + 5(4)x + 16 \rightarrow \\ 6x^2 + 8x + 12x + 16 &= 6x^2 + 20x + 16 \rightarrow \\ 6x^2 + 20x + 16 &= 6x^2 + 20x + 16\end{aligned}$$

4. **(D)** Manipulate the equation to isolate for C , the molar concentration.

$$\Psi_s = -iCRT \rightarrow$$

Divide both sides by $-iRT \rightarrow$

$$C = -\frac{\Psi_s}{iRT}$$

5. **(C)** The function is a parabola that is facing downwards, given the -16 in front of the largest power term, t^2 . The maximum value for the height will therefore be at the parabola's vertex. To find the value of t at the vertex, the easiest way to proceed is to use the formula $x = -\frac{b}{2a}$. The constant a is -16 , and the constant b is 20 . Instead of x , we can use the variable t :

$$x = -\frac{b}{2a} \rightarrow$$

$$t = -\frac{b}{2a} \rightarrow$$

$$t = -\frac{20}{2(-16)} \rightarrow$$

$$t = \frac{20}{32} = 0.625$$

6. **(A)** FOIL the left-hand side of the equation so you can identify the components that correspond to the values on the right side:

$$(2x - 3y)(4x + y) = 8x^2 + axy + bxy - 3y^2 \rightarrow$$

$$8x^2 + 2xy - 12xy - 3y^2 = 8x^2 + axy + bxy - 3y^2 \rightarrow$$

Subtract the $8x^2$ and $-3y^2$ from both sides $\rightarrow$

$$2xy - 12xy = axy + bxy \rightarrow$$

Divide both sides by $xy \rightarrow$

$$2 - 12 = a + b \rightarrow$$

$$-10 = a + b$$

So, -10 equals the sum of a and b .

7. **(C)** The y value of the highest point on the function is 10, and the y value of the lowest point on the function is 2. So, the difference between the maximum and the minimum of the function will be $10 - 2 = 8$.
8. **(A)** Distribute the k to see potential equivalences:

$$k(ax^2 + bx + 5) = 6x^2 - 21x + 15 \rightarrow$$

$$kax^2 + kbx + 5k = 6x^2 - 21x + 15$$

Based on this, we have three different equivalences:

$$ka = 6, kb = -21, \text{ and } 5k = 15$$

Since we do not know the values of a and b , the easiest way to solve this is to solve $5k = 15$:

$$\begin{aligned} 5k &= 15 \rightarrow \\ \frac{5k}{5} &= \frac{15}{5} \rightarrow \\ k &= 3 \end{aligned}$$

9. **(C)** Plug $x + n$ into $f(x) = x^2 - 4$, then make this expression equivalent to $x^2 + 6x + 5$ to solve for n :

$$\begin{aligned} (x + n)^2 - 4 &\rightarrow \\ (x + n)(x + n) - 4 &\rightarrow \\ x^2 + 2xn + n^2 - 4 &= x^2 + 6x + 5 \end{aligned}$$

Given this equation, the easiest way to proceed is probably to set $2xn$ equal to $6x$ and solve for n :

$$\begin{aligned} 2xn &= 6x \rightarrow \\ 2n &= 6 \rightarrow \\ n &= 3 \end{aligned}$$

10. **(A)** Solve this by making an equation that has only x and z in it, eliminating the y .

$$\begin{aligned} 2y &= 3x - 4 \rightarrow \\ y &= \frac{3}{2}x - 2 \rightarrow \end{aligned}$$

Substitute this for y in the other equation:

$$\begin{aligned} \left(\frac{3}{2}x - 2\right) + 1 &= z - 5 \rightarrow \\ \frac{3}{2}x - 1 &= z - 5 \rightarrow \\ \frac{3}{2}x &= z - 4 \rightarrow \\ x &= \frac{2}{3}z - \frac{8}{3} \end{aligned}$$

Problem Solving and Data Analysis

This section contains comprehensive review and strategies for all of the concepts below:

Problem Solving and Data Analysis—Measures of Center

- Mean
- Median
- Mode
- Outliers
- Range and Standard Deviation

Problem Solving and Data Analysis—Unit Conversion

Problem Solving and Data Analysis—Surveys

- Sampling Methods
- Drawing Conclusions

Problem Solving and Data Analysis—Percentages

- Percentage Essentials
- Percent Increase and Decrease
- Simple Interest
- Compound Interest

Problem Solving and Data Analysis—Graphs and Data Interpretation

- Probability
- Graph Interpretation
- Scatterplots
- Histograms
- Tables
- Box Plots

- Linear and Exponential Growth
- Interpreting Constants and Functions

Problem Solving and Data Analysis—Measures of Center

The SAT Math will assess your understanding of different ways of calculating averages. Mean, median, and mode are the three major concepts to know.

Mean

$$\frac{\text{Sum of Items}}{\text{Number of Items}} = \text{Mean}$$

This is what you typically would do if asked to find the average of a set of terms.

▸ Example 1

What is the mean of the numbers 2, 4, 7, 12, 14, and 18?

✓ Solution

$$\frac{\text{Sum of Items}}{\text{Number of Items}} = \frac{2 + 4 + 7 + 12 + 14 + 18}{6} = \frac{57}{6} = 9.5$$

▸ Example 2

There are four numbers in a set: 4, 10, 12, and x . If the mean of the four terms is 10, what is the value of x ?

✓ Solution

Set up an equation for the mean of the set, keeping x as an unknown value:

$$\frac{\text{Sum of Items}}{\text{Number of Items}} = \text{Mean} \rightarrow$$

$$\frac{4 + 10 + 12 + x}{4} = 10 \rightarrow$$

$$\frac{26 + x}{4} = 10 \rightarrow$$

$$26 + x = 40 \rightarrow$$

$$x = 14$$

So, the value of x is 14.

Median

The *middle term* of a set of numbers when arranged from *least to greatest*. (If there is an even number of terms and the two middle terms are different, take the mean of the two middle terms to find the median of the set of numbers.)

➤ Example 1

What is the median of this set of numbers? {1, 3, 6, 9, 12, 17, 22}

✓ Solution

The numbers are already arranged from least to greatest, so find the middle term. In this case, since there are seven terms, the fourth term will be the middle term. The answer is therefore 9.

➤ Example 2

What is the median of this set of numbers? {6, 4, 3, 10, 13, 14}

✓ Solution

First, arrange the numbers from least to greatest.

$$\{3, 4, 6, 10, 13, 14\}$$

Notice that there are six numbers, and the two middle numbers are different. So, to find the median of this set, we must take the mean of 6 and 10:

$$\frac{6 + 10}{2} = \frac{16}{2} = 8$$

Mode

This is the *most frequent* number in a set. (Note that if a set of numbers has each number appear just once, there will be zero modes. If a set of numbers has two or more numbers tied for appearing the most times, it will have multiple modes.)

➤ Example

What is the mode of this set of numbers? {1, 5, 3, 9, 2, 4, 8, 9}

✓ Solution

Every number in the set appears just once, except for 9, which appears twice. Therefore, 9 is the mode of the set.

Outliers

A number in a set that is quite different from the other values in the set.

➤ Example

Which value in this set would be considered an outlier? {2, 3, 5, 6, 8, 240}

✓ Solution

While the first five numbers in the set are rather small, 240 is far larger than the others and would be considered an outlier.

Practice

What is the mean of this set of numbers? {1, 4, 9, 10}

What is the median of this set of numbers? {2, 4, 8, 9, 12}

What is the mode of this set of numbers? {4, 5, 7, 8, 4, 2, 9}

There are a total of four numbers in a set. The sum of the three smallest terms is 80. If the average of the set of four numbers is 30, what is the value of the largest number in the set?

What is the median of 4, 3, 8, 10, 20, and 12?

In this set of numbers, {1, 2, 4, 5, 5, 20, 56}, is the mean, median, or mode the largest?

If an outlier is added to a set of numbers, would it most likely affect the mode, median, or mean of the set?

Solutions

$$\frac{\text{Sum of Items}}{\text{Number of Items}} = \text{Mean} \rightarrow$$
$$\frac{1 + 4 + 9 + 10}{4} = \frac{24}{4} = 6$$

There are five terms in the set, with 8 being the middle term since they are already in order from least to greatest. Therefore, 8 is the median.

All the numbers appear just once, except for 4, which appears twice. Thus, 4 is the mode of the set.

Calculate the value of the missing term, which we can call x , using the mean formula. Since the smallest three terms add up to 80, we can add 80 and x to get the sum of the items.

$$\frac{\text{Sum of Items}}{\text{Number of Items}} = \text{Mean} \rightarrow$$
$$\frac{80 + x}{4} = 30 \rightarrow$$
$$80 + x = 120 \rightarrow$$
$$x = 40$$

Put the numbers in order from least to greatest:

$$3, 4, 8, 10, 12, 20$$

There are six terms and the third and fourth terms are different. So, to calculate the median of the set, we must take the mean of 8 and 10:

$$\frac{8 + 10}{2} = \frac{18}{2} = 9$$

Rather than calculating the individual values of the median, mean, and mode for the set—{1, 2, 4, 5, 5, 20, 56)—you can take a shortcut. Since the set has two numbers, 20 and 56, that are far greater than the others, they will cause the mean to be greater than the mode and median.

Alternatively, you can calculate the individual values of the mean, median, and mode:

The mean is $\frac{1 + 2 + 4 + 5 + 5 + 20 + 56}{7} \approx 13.3$

The median is 5 since the fourth number out of the set of seven numbers is 5.

The mode is 5 since it appears most frequently.

So, however you find it, the mean will be the greatest.

An outlier would most likely affect the *mean* of a set of numbers, since it would be included in the mathematical calculation of the average. It is possible that an outlier would not change the median, so long as the middle value remained the same. It is also possible that an outlier would not change the mode, so long as the outlier was not repeated.

Range and Standard Deviation

To analyze properties of data sets, understand the concepts of range and standard deviation.

Range is the difference between the smallest and largest values in a set of data.

➤ Example

What is the range of this set of numbers? {1, 3, 7, 9, 15}

✓ Solution

Take the difference between the largest and smallest values in the set to find the range:

$$15 - 1 = 14$$

Therefore, the range is 14.

Standard deviation indicates how significant the *spread* of the data in a set is.

Mathematically, it is calculated as:

$$\sqrt{\text{Average of the squared distances of the data points from their mean}}$$

Fortunately, instead of having you calculate the precise value of the standard deviation, the SAT tests your understanding of the *concept* of standard deviation.

If the standard deviation in a set of numbers is *small*, there is little variation in the data.

If the standard deviation in a set of numbers is *large*, there will be greater variation in the data.

► Example

Which of the following sets will have a greater standard deviation?

Set X: {5, 6, 7, 8, 9}

Set Y: {1, 20, 40, 80, 95}

✓ Solution

Set Y will have a greater standard deviation because the numbers deviate more greatly from its mean. The values in Set X are much more closely clustered around one another.

Practice

What is the range of this set of numbers? {-5, 3, 8, 20, 28}

Which of the following numbers, if added to the set {2, 5, 8, 11, 15} would most likely *decrease* its standard deviation?

- A) 2
- B) 5
- C) 8
- D) 15

What positive number could be added to the set {2, 4, 8, 9, 18} to double its range?

Would a sample of ages among students in an elementary school or a sample of ages among residents of a city more likely have a greater standard deviation?

Solutions

Find the difference between the least and greatest values in the set to calculate the range:

$$28 - (-5) = 28 + 5 = 33$$

Since the standard deviation is calculated by the

$$\sqrt{\text{Average of the squared distances of the data points from their mean}}$$

if you add a data point that *is equal* to the mean, that would certainly reduce the standard deviation of the set. Since the number 8 is equal to the mean, adding it would ensure that the average of the squared distances of the data points from their mean would decrease. So, the answer is **(C)**, 8.

First, let's calculate the original range of the set by subtracting the least value from the greatest value:

$$18 - 2 = 16$$

Now, let's double this range:

$$16 \times 2 = 32$$

Since the question asks us to find a *positive* number that could be added to the set that would cause it to have this doubled range of 32, we can add the least value to this range to find the needed value to be added:

$$2 + 32 = 34$$

So, if 34 were added to the set, the range of the set would double.

Standard deviation provides a measure of the *spread* of data points—the greater the spread, the greater the standard deviation. In an elementary school, the ages of students would most likely be clustered around lower ages, like 5–12. In a town, the residents would range in age from very young to very old. So, it is more likely that the standard deviation of ages in the town would be greater.

Practice Drill—Measures of Center

(If timing, take about 12 minutes to complete.)

{2, 7, 9, x , 12, 15}

Consider the data set above. What must the value of x be so that the median of the data set is 10?

- A) 9
- B) 10
- C) 11
- D) 12

What is the mean of the fractions $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$?

- A) $\frac{1}{5}$
- B) $\frac{3}{13}$
- C) $\frac{1}{4}$
- D) $\frac{1}{3}$

For a fundraiser, 8 students sold fewer than 5 coupon books. Ten students sold at least 5 and no more than 10 coupon books. Fifteen students sold 11 or more coupon books. Assuming that the previously mentioned students represent the entire set of students, which of the following could have been the median number of coupon books for this group based on this information?

- A) 4
- B) 8
- C) 10.5
- D) 12

Number of Apps on a Phone (Range of Values)	How Many Students with Number of Apps in Range
0–5	2
6–10	5
11–15	7
16–20	4
More than 20	2

4. Twenty students were asked how many apps they had on their phones. The results are provided in the table above. If two students are added to this set, one of whom has just 3 apps and the other of whom has 26 apps, how would the median number of apps per student be affected?
 - (A) Increases
 - (B) Decreases
 - (C) Remains the same
 - (D) Cannot determine from the given information
5. Five students purchased a computer that costs \$1,000. The students split the cost equally among themselves. In order for the share per person to

decrease by \$75, how many additional students would they need to share in the purchase, given that they would all share the cost equally?

- (A) 3
- (B) 4
- (C) 6
- (D) 8

6. $\{1, 4, 5, 5.5, 6, 6.5, 9.5\}$

In the above set of numbers, the two values that are the greatest outliers from the others are removed. Once these two values are removed, what is the range of the remaining values?

- (A) 2.5
- (B) 3
- (C) 7
- (D) 8.5

7. The maximum weight a certain elevator can hold is 3,000 pounds. For a group of 15 people to go on the elevator, what is the maximum the mean of their weights could be?

- (A) 170 pounds
- (B) 190 pounds
- (C) 200 pounds
- (D) 240 pounds

8. A student had scores of 80, 90, 85, 100, 75, and 85 on his tests in a history class. If the teacher threw out his worst test grade, which of these values describing the set of his scores would change?

- (A) Mean
- (B) Median

- (C) Mode
- (D) None of the above

9. Jake has a smartwatch that helps him keep track of the number of steps he takes each day. On Monday, he walked 8,000 steps; Tuesday, 13,000; Wednesday, 7,000; and Thursday, 8,500. Which inequality expresses the number of steps, n , that Jake could walk on Friday so that the average for the 5 days is at least 10,000 steps per day?

- (A) $\frac{8,000 + 13,000 + 7,000 + 8,500 + n}{5} \leq 10,000$
- (B) $\frac{8,000 + 13,000 + 7,000 + 8,500 + n}{5} \geq 10,000$
- (C) $\frac{8,000 + 13,000 + 7,000 + 8,500}{n} \geq 10,000$
- (D) $\frac{8,000 + 13,000 + 7,000 + 8,500 + n}{4} \leq 10,000$

10. The heights in inches of 10 people in a room is given below:

$\{60, 49, 72, 75, 54, 58, 76, 44, 51, 53, 62\}$

If someone new comes in to the room and has a height of 74 inches, which of these is an accurate statement about the range and standard deviation of the set?

- (A) Only the standard deviation would change.
- (B) Only the range would change.
- (C) Both the standard deviation and range would change.
- (D) Neither the standard deviation nor range would change.

Solutions

C) There are 6 numbers in this set, so the median will be the average of the two middle terms. If the two middle terms were 7 and 9, the median would be 8; if the two middle terms were 9 and 12, the median would be 10.5. So, x must be between 9 and 12 and must average out to 10. You can therefore solve for x by solving the following equation:

$$\begin{aligned}\frac{x+9}{2} &= 10 \rightarrow \\ x+9 &= 20 \rightarrow \\ x &= 11\end{aligned}$$

Thus, 11 would be the value of x that would cause the set of numbers to have a median of 10.

C) First, modify all the fractions so they have the same denominator, 12:

$$\frac{1}{3} = \frac{4}{12}, \frac{1}{4} = \frac{3}{12}, \text{ and } \frac{1}{6} = \frac{2}{12}$$

Then, add the fractions together and divide by 3 to solve for the mean.

$$\frac{\left(\frac{4}{12} + \frac{3}{12} + \frac{2}{12}\right)}{3} = \frac{\left(\frac{9}{12}\right)}{3} = \frac{3}{12} = \frac{1}{4}$$

B) Let us find the total number of students in the set: $8 + 10 + 15 = 33$. The middle value will be the 17th value, since there would be 16 values less than it and 16 values greater than it.

Since there are 8 who sold fewer than 5, and 10 students who sold between 5–10 books, the 17th term if the terms were placed in order from least to greatest would be within the range of 5–10 books. The answer is therefore 8 because it is the only number within this range.

C) Because there are 20 students, the initial median will be the average of the 10th and 11th terms. The 10th and 11th terms will both fall between the range of 11–15 apps based on the table. If a student with 3 apps and a student with 26 apps were added to this set, they would balance each other out and there would be no impact on the median of the set. Therefore, the median of the set would remain the same.

A) First, let's find the original cost per student by calculating the mean when there are 5 students and a total price of \$1,000:

$$\frac{1,000}{5} = 200$$

So, the initial share per student is \$200. If the amount of the share per student is to decrease by \$75, the new share per student would be $200 - 75 = 125$ dollars. Set up a new calculation of the mean to solve for the number of students, x , who would need to be added to the initial 5 students to get a new average of \$125:

$$\begin{aligned}\frac{1,000}{5+x} &= 125 \rightarrow \\ 1,000 &= 125(5+x) \rightarrow \\ 1,000 &= 625 + 125x \rightarrow \\ 375 &= 125x \rightarrow \\ x &= 3\end{aligned}$$

Therefore, if 3 students were added to the total, there would be a total of 8 students who could share the cost at an equal amount of \$125 a person.

A) The two greatest outliers in the set are the values that are farthest from the mean: 1 and 9.5. If we remove these values, the new set is {4, 5, 5.5, 6, 5.5}. To calculate the range of this new set, subtract the smallest value from the largest value:

$$6.5 - 4 = 2.5$$

C) Since the maximum weight is 3,000 pounds, divide 3,000 by 15 to find the mean under these conditions:

$$\frac{3,000}{15} = 200$$

A) Let's see how the mean, median, and mode will be affected by this change of removing the score of 75, the worst test grade, from the set. The original set, when the numbers are put in order from least to greatest, is {75, 80, 85, 85, 90, 100}. The new set would be {80, 85, 85, 90, 100}.

Original Set: {75, 80, 85, 85, 90, 100}	New Set: {80, 85, 85, 90, 100}
Original Mean: $\frac{75 + 80 + 85 + 85 + 90 + 100}{6} = \frac{515}{6} \approx 85.83$	New Mean: $\frac{80 + 85 + 85 + 90 + 100}{5} = \frac{440}{5} = 88$
Original Median: {75, 80, 85, 85, 90, 100}: The third and fourth terms are both 85, so the median is 85.	New Median: {80, 85, 85, 90, 100}: The third term is 85, so the median is still 85.
Original Mode: 85 is the most frequent term.	New Mode: 85 is still the most frequent term.

Therefore, only the mean of the set will change. If you notice that the mode and median will not be affected, you can see that just the mean will be affected since removing one value that is less than the original mean will inevitably change the mean of the set.

B) This is a question where looking ahead to the answers can help, since the answers show that you just need to set up the calculation, rather than doing it all the way to completion. Be careful to avoid careless mistakes, especially with the direction of the inequality sign. Jake wants to average *at least* 10,000 steps per day, so the average number of steps must be *greater than or equal to* 10,000: $average \geq 10,000$. The average is calculated by adding the known values together and treating n as the one day that is not known. There are 5 total days including n . Therefore, the calculation will be:

$$\frac{8,000 + 13,000 + 7,000 + 8,500 + n}{5} \geq 10,000$$

A) The range of the set is the difference between the least and greatest terms in the set: $76 - 44 = 32$. The range will not change if 74 were added to the set, since the values of the least and greatest numbers in the set would remain at 44 and 76.

The standard deviation of the set *would* change since the variation of the numbers from the mean will be different with the relatively large value of 74 added to the group. Therefore, the answer is **(A)**.

Problem Solving and Data Analysis—Unit Conversion

SAT Math Strategy

Use proportions to solve simple unit conversions, and use unit cancellation to solve more complex unit conversions.

If an SAT Math problem involves a straightforward calculation between just two units, you can probably solve it using a proportion.

► Example 1

There are 5,280 feet in 1 mile. If Susan runs 3.5 miles, how many feet did she run?

✓ Solution

Set up a proportion to solve for the x number of feet that Susan ran in the race. It is easier if you place the variable for which you are solving *in the numerator* so the algebra is less complex.

$$\frac{5,280 \text{ feet}}{1 \text{ mile}} = \frac{x \text{ feet}}{3.5 \text{ miles}} \rightarrow$$

Multiply both sides by 3.5 miles to solve for $x \rightarrow$

$$\frac{5,280 \text{ feet}}{1 \text{ mile}} \times 3.5 \text{ miles} = \frac{x \text{ feet}}{3.5 \text{ miles}} \times 3.5 \text{ miles} \rightarrow$$

Cancel out the miles units and the 3.5 on the right side $\rightarrow$

$$\frac{5,280 \text{ feet}}{1 \text{ mile}} \times \cancel{3.5 \text{ miles}} = \frac{x \text{ feet}}{\cancel{3.5 \text{ miles}}} \times \cancel{3.5 \text{ miles}} \rightarrow$$

$$5,280 \text{ feet} \times 3.5 = x$$

$$18,480 \text{ feet} = x$$

► Example 2

Ahmad has 300 U.S. dollars that he wants to convert to euros. His bank offers an exchange rate of 0.83 euros for 1 U.S. dollar. Assuming there are

no fees or taxes, if Ahmad converts all 300 of his dollars to euros, how many euros will he have?

✓ Solution

Set up a proportion to solve for euros:

$$\frac{0.83 \text{ Euros}}{1 \text{ Dollar}} = \frac{x \text{ Euros}}{300 \text{ Dollars}} \rightarrow$$

Multiply both sides by 300 dollars to solve for $x \rightarrow$

$$\frac{0.83 \text{ Euros}}{1 \text{ Dollar}} \times 300 \text{ Dollars} = \frac{x \text{ Euros}}{300 \text{ Dollars}} \times 300 \text{ Dollars} \rightarrow$$

Cancel out the dollars units and the 300 on the right side $\rightarrow$

$$\frac{0.83 \text{ Euros}}{1 \text{ Dollar}} \times \cancel{300 \text{ Dollars}} = \frac{x \text{ Euros}}{\cancel{300 \text{ Dollars}}} \times \cancel{300 \text{ Dollars}} \rightarrow$$

$$0.83 \text{ Euros} \times 300 = x$$

$$249 \text{ Euros} = x$$

So, Ahmad would have 249 euros after the currency conversion.

➤ Example 3

Emily took a bike ride that was 8 kilometers long. Given that there are approximately 0.6214 miles in a kilometer and 5,280 feet in a mile, approximately how many *feet* was Emily's bike ride?

✓ Solution

Ultimately, your unit conversion will end with feet. Use *dimensional analysis* or the *unit-factor method* (different ways of labelling the same idea) to cancel out units and end with the desired unit. The ride is 8 kilometers, and there are 0.6214 miles in a kilometer and 5,280 feet in a mile. So, set up the unit conversion like this:

$$8 \text{ kilometers} \times \frac{0.6214 \text{ miles}}{1 \text{ kilometer}} \times \frac{5,280 \text{ feet}}{1 \text{ mile}} \rightarrow$$

Cancel out units that are in both the numerator and denominator:

$$8 \text{ kilometers} \times \frac{0.6214 \text{ miles}}{1 \text{ kilometer}} \times \frac{5,280 \text{ feet}}{1 \text{ mile}} \rightarrow$$
$$8 \times 0.6214 \times 5,280 \approx 26,248 \text{ feet}$$

► Example 4

Timothy has 1 gallon of milk. There are approximately 3.785 liters in 1 gallon. How many milliliters of milk does Timothy have?

✓ Solution

While the SAT will give you many conversion ratios, there are some that it will be helpful to memorize:

- 12 inches in 1 foot
- 60 seconds in 1 minute
- 60 minutes in 1 hour
- 100 centimeters in 1 meter (Prefix *centi* means “one-hundredth.”)
- 1,000 millimeters in 1 meter (Prefix *milli* means “one-thousandth.”)
- 1,000 milliliters in 1 liter (Prefix *milli* means “one-thousandth.”)
- 1,000 meters in 1 kilometer (Prefix *kilo* means “thousand.”)

In this problem, we will ultimately end up with milliliters. Use *dimensional analysis* or the *unit-factor method* to cancel out units and end with the desired unit of milliliters:

$$1 \text{ gallon} \times \frac{3.785 \text{ liters}}{1 \text{ gallon}} \times \frac{1,000 \text{ milliliters}}{1 \text{ liter}} \rightarrow$$
$$1 \text{ gallon} \times \frac{3.785 \text{ liters}}{1 \text{ gallon}} \times \frac{1,000 \text{ milliliters}}{1 \text{ liter}} \rightarrow$$
$$1 \times 3.785 \times 1,000 = 3,785 \text{ milliliters}$$

Practice Drill—Problem Solving and Data Analysis: Unit Conversion

(If timing, take about 12 minutes to complete.)

Approximately how many kilometers are in a marathon, given that a marathon is 26.22 miles and that there are 1.609 kilometers in a mile?

- A) 16.3 km
- B) 27.8 km
- C) 42.2 km
- D) 74.5 km

The Empire State Building in New York City has a height of 1,250 feet, while if its pinnacle is included, it has a height of 1,454 feet. What is the height of just the pinnacle to the nearest whole meter given that there are 3.28 feet in 1 meter?

- A) 62 meters
- B) 85 meters
- C) 204 meters
- D) 669 meters

If 1 ounce of platinum sells for \$700 and 1 ounce of gold sells for \$1,700, how much more expensive would a pound of gold be than a pound of platinum, given that there are 16 ounces in a pound?

- A) \$7,000
- B) \$16,000
- C) \$27,200
- D) \$38,400

A barrel has a volume of 42 gallons. If you have 105 gallons of a liquid, what would be the equivalent number of barrels of the liquid?

- A) 0.4 barrels
- B) 1.5 barrels
- C) 2.5 barrels
- D) 6.8 barrels

The depth in *feet* of water, D , in a lake x days after it was dammed is given by this equation:

$$D = 0.5x + 30$$

Based on this equation, how many *inches* does the depth of the water in the lake increase each day?

- A) 0.5
- B) 3.5
- C) 6
- D) 30

In 1875, Matthew Webb was the first person to successfully swim across the English Channel. He took about 22 hours to make the roughly 22-mile swim. Given that there are about 1.61 kilometers in a mile, what was his approximate average swimming speed for this trip in kilometers per hour?

- A) 0.62 km/hour
- B) 1.61 km/hour
- C) 35 km/hour
- D) 44 km/hour

The speed of sound as it travels through rubber is approximately 60 meters per second. How fast does sound travel through rubber in kilometers per hour?

- A) Approximately 216 km/hour
- B) Approximately 642 km/hour

- C) Approximately 36,000 km/hour
D) Approximately 216,000 km/hour

Fill-In Practice: Write your answer in the underlined blank under each question.

8. Mark rides his bicycle 30 feet in 2 seconds. At that rate, how many feet would he travel in 1 minute on his bicycle?

Answer: _____

9. The key on a map indicates that 1 inch on the map corresponds to $\frac{1}{2}$ of a mile in actual distance. If the distance on the map between two points is 6 inches, how many miles apart would they be in real life?

Answer: _____

10. The number of cloves in a head of garlic is between 10 and 12. If a recipe called for 3.5 heads of garlic, what would a possible value for the number of cloves that would meet this requirement?

Answer: _____

Solutions

- C) Use a proportion to solve for the number of kilometers in a marathon:

$$\frac{1.609 \text{ kilometers}}{1 \text{ mile}} = \frac{x \text{ kilometers}}{26.22 \text{ miles}} \rightarrow$$

Cross Multiply:

$$\frac{1.609 \text{ kilometers}}{1 \text{ mile}} \times 26.22 \text{ miles} = x$$

Cancel out the miles:

$$\frac{1.609 \text{ kilometers}}{1 \text{ mile}} \times 26.22 \text{ miles} = x \rightarrow$$

$$1.609 \text{ kilometers} \times 26.22 \approx 42.2 \text{ kilometers}$$

A) First, determine the height of the pinnacle by itself by subtracting the height of just the building from the height of the building and pinnacle combined:

$$1,454 - 1,250 = 204$$

Then, set up a proportion to solve for the height of the pinnacle in meters:

$$\frac{1 \text{ meter}}{3.28 \text{ feet}} = \frac{x \text{ meters}}{204 \text{ feet}} \rightarrow$$

$$\frac{1 \text{ meter}}{3.28 \text{ feet}} \times 204 \text{ feet} = x \rightarrow$$

$$\frac{1 \text{ meter}}{3.28 \text{ feet}} \times 204 \text{ feet} = x \rightarrow$$

$$\frac{204}{3.28} \approx 62 \text{ meters}$$

B) First, recognize that we need to convert the difference between the price of gold and platinum:

$$1,700 - 700 = 1,000$$

Then, multiply \$1,000 by 16 ounces to find the total difference in price per pound:

$$1,000 \times 16 = 16,000$$

C) Use a proportion to solve this problem:

$$\frac{1 \text{ barrel}}{42 \text{ gallons}} = \frac{x \text{ barrels}}{105 \text{ gallons}} \rightarrow$$

$$\frac{1 \text{ barrel}}{42 \text{ gallons}} \times 105 \text{ gallons} = x \rightarrow$$

$$\frac{1 \text{ barrel}}{42 \text{ gallons}} \times 105 \text{ gallons} = x \rightarrow$$

$$\frac{105}{42} = 2.5 \text{ barrels} = x$$

C) The equation is written in slope-intercept form, so to see how much the depth is increasing in feet each day, look at the slope—0.5. Then, convert .5 feet to inches. Since there are 12 inches in a foot, multiply 0.5 feet by 12 inches to get the converted number of inches:

$$0.5 \times 12 = 6$$

So, each day the depth of water increases by 6 inches.

B) We want to have the units result in kilometers per hour, so use unit cancellation to determine this speed:

$$\frac{22 \text{ miles}}{22 \text{ hours}} \times \frac{1.61 \text{ kilometers}}{1 \text{ mile}} \rightarrow$$

$$\frac{22 \cancel{\text{ miles}}}{22 \text{ hours}} \times \frac{1.61 \text{ kilometers}}{1 \cancel{\text{ mile}}} \rightarrow$$

$$\frac{22}{22} \times 1.61 = 1.61 \frac{\text{km}}{\text{hr}}$$

Alternatively, you could have noticed that the speed is 1 mile per hour, so just multiply 1 mile per hour by 1.61.

A) Use unit-cancellation to solve for the speed in kilometers per hour:

$$\frac{60 \text{ meters}}{1 \text{ second}} \times \frac{1 \text{ kilometer}}{1,000 \text{ meters}} \times \frac{3,600 \text{ seconds}}{1 \text{ hour}} =$$

$$\frac{60 \cancel{\text{ meters}}}{1 \cancel{\text{ second}}} \times \frac{1 \text{ kilometer}}{1,000 \cancel{\text{ meters}}} \times \frac{3,600 \cancel{\text{ seconds}}}{1 \text{ hour}} \approx 216 \frac{\text{km}}{\text{hr}}$$

900) Use unit-cancellation to solve for the distance he will travel in 1 minute:

$$\frac{30 \text{ feet}}{2 \text{ seconds}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \rightarrow$$

$$\frac{30 \text{ feet}}{2 \cancel{\text{ seconds}}} \times \frac{60 \cancel{\text{ seconds}}}{1 \text{ minute}} \rightarrow$$

$$\frac{30 \times 60}{2} = 900 \frac{\text{feet}}{\text{minute}}$$

3) Use a proportion to solve for the miles:

$$\frac{0.5 \text{ mile}}{1 \text{ inch}} = \frac{x \text{ miles}}{6 \text{ inches}} \rightarrow$$

$$\frac{0.5 \text{ mile}}{1 \text{ inch}} \times 6 \text{ inches} = x \rightarrow$$

$$\frac{0.5 \text{ mile}}{1 \text{ inch}} \times 6 \text{ inches} = x \rightarrow$$

$$0.5 \times 6 = 3 \text{ miles} = x$$

Any number greater than or equal to 35 and less than or equal to 42.)

Find the lower number by multiplying 10 by 3.5, and the greater number by multiplying 12 by 3.5:

$$10 \times 3.5 = 35$$

$$12 \times 3.5 = 42$$

So, the range of garlic cloves would be between 35 and 42, inclusive (meaning including 35 and 42 also).

Problem Solving and Data Analysis—Percentages

Percentage Essentials

Percent calculations are heavily emphasized on the SAT Math. To be successful on percent problems, there are several key formulas it is helpful to memorize.

$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent}$
--

In most situations you will come across, the “part” will be the smaller number and the “whole” will be the larger number.

➤ Example 1

If there are 200 students in a graduating class, and 40 of them graduated with honors, what percent of students in the class graduated with honors?

✓ Solution

The “part” is 40, and the “whole” is 200. Plug these values into the above formula to calculate the percent:

$$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent} \rightarrow$$
$$\frac{40}{200} \times 100 = 0.2 \times 100 = 20\%$$

► Example 2

If 60% is considered a passing grade on a particular test, and the test has 30 questions, how many questions would a test taker have to answer correctly to pass the test?

✓ Solution

Use 60% as the “percent,” and use 30 as the “whole,” since 30 represents the total number of questions. Then, use the percent formula to solve for the “part”—the number of questions that must be answered correctly to pass:

$$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent} \rightarrow$$
$$\frac{\text{Part}}{30} \times 100 = 60 \rightarrow$$
$$\text{Part} = \frac{60 \times 30}{100} = \frac{1,800}{100} = 18$$

So, one would need to answer at least 18 questions correctly to pass the test.

► Example 3

If 30% of the cars in a parking lot have bumper stickers, what fraction of the cars in the parking lot would have bumper stickers?

✓ Solution

Manipulate the percent formula so we can convert this situation to a fraction:

$$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent} \rightarrow$$

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}$$

Use 30 as the percent, then reduce the fraction:

$$\frac{30}{100} = \frac{3}{10}$$

So, $\frac{3}{10}$ of the cars in the parking lot have bumper stickers.

Example 4

Michal took a test that had 20 points, and she earned 2 points of extra credit in addition to a perfect score on the test. What was her percentage score?

Solution

In this situation, the “part” will be 22 to include the perfect score and the extra credit. The “whole” will be 20, given that total number of points on the test. So, calculate her percentage scores as follows:

$$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent} \rightarrow$$

$$\frac{22}{20} \times 100 = 110\%$$

For problems like these, it can be helpful to know the conversions of fractions to percentages off the top of your head:

$10\% = \frac{1}{10}$
$25\% = \frac{1}{4}$
$50\% = \frac{1}{2}$
$75\% = \frac{3}{4}$

The part/whole formula will always work, so you can stick to that approach if you are more comfortable. There are, however, a number of ways to work with percentages more efficiently depending on the type of problem.

When asked to find the percent of a number, take the percent and move the decimal point *to the left two spots and then multiply*.

➤ Example 1

What is 75% of 500?

✓ Solution

Rather than using the previous percent formula to solve, start by taking the 75% and moving the decimal point to the left by two spots:

$$75.0 \rightarrow 0.75$$

Now multiply by 500:

$$500 \times 0.75 = 375$$

So, 375 represents 75% of 500.

➤ Example 2

What is 4% of 20?

✓ Solution

Move the decimal point of 4% to the left by two spots:

$$4.0 \rightarrow 0.04$$

Now multiply by 20:

$$0.04 \times 20 = 0.8$$

So, 0.8 is 4% of 20.

► Example 3

What is 150% of 70?

✓ Solution

Move the decimal point of 150 to the left by two spots:

$$150.0 \rightarrow 1.5$$

Then, multiply by 70:

$$70 \times 1.5 = 105$$

Practice

What is 40% of 240?

Two out of every five students at a school pack their lunches each day. What percent of students at the school pack their lunches each day?

What is 5% of 125?

With extra credit, Chris had a 105% as his grade for the semester in a class. If he scored 100% on every assignment throughout the semester, and there were a total of 400 points from all the regular assignments (not including extra credit), how many points in extra credit did he get that semester?

If 60 employees in a business work part-time, and these employees represent 30% of all the employees at the business, how many total employees does the business have?

Solutions

$$0.4 \times 240 = 96$$

$$\frac{2}{5} \times 100 = 40\%$$

$$0.05 \times 125 = 6.25$$

Just take 5% of the total points from the regular assignments to find the points in extra credit he earned:

$$0.05 \times 400 = 20$$

Plug in values to the general percentage formula to solve for the total number of employees:

$$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent} \rightarrow$$

$$\frac{60}{\text{Whole}} \times 100 = 30 \rightarrow$$

Cross multiply $\rightarrow$

$$\frac{60}{30} \times 100 = \text{Whole} \rightarrow$$

$$200 = \text{Whole}$$

So, the business has a total of 200 employees.

Percent Increase and Decrease

To find the percent by which something has increased or decreased, use this formula, being careful on what you plug in for the “original” and “new” values:

$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change}$

If the value of the percent change is positive, the change is increasing. If the value is negative, the change is decreasing.

► Example 1

Melanie's height is 40 inches. A year later, her height is 44 inches. By what percent did her height increase?

✓ Solution

The original value is 40, and the new value is 44, so the calculation is

$$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change}$$
$$\frac{44 - 40}{40} \times 100 = \frac{4}{40} \times 100 = 10\%$$

So, Melanie's height has increased by 10%.

> Example 2

William has \$800 in his checking account. If he spends \$200 out of his checking account, by what percent did his checking account decrease?

✓ Solution

The new value is \$600 (\$800 – \$200), and the original value is \$800, so the calculation is

$$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change} \rightarrow$$
$$\frac{600 - 800}{800} \times 100 = \text{Percent Change} \rightarrow$$
$$\frac{-200}{800} \times 100 = -25\%$$

Since this value is *negative*, there has been a 25% *decrease* in his checking account.

What are easy ways to calculate the new amount if we know the percent by which something has increased or decreased?

When the percentage is expressed as a decimal, r :

$$\text{Increased Total} = (\text{Original Amount}) \times (1 + r)$$

$$\text{Decreased Total} = (\text{Original Amount}) \times (1 - r)$$

➤ Example 1

If the cost of a gallon of milk, originally at \$2 a gallon, increased by 20%, what would be the new cost of a gallon of milk?

✓ Solution

Use the increased total formula, plugging in 2 for the original amount, and using 0.20 as the r since we must express 20% as a decimal:

$$\begin{aligned}\text{Increased Total} &= (\text{Original Amount}) \times (1 + r) \rightarrow \\ \text{New Price} &= (2) \times (1 + 0.2) = 2 \times 1.2 = 2.40\end{aligned}$$

So, the new price of a gallon of milk would be \$2.40.

➤ Example 2

If a toy is originally \$30 and is on sale for 40% off, what would its new price be?

✓ Solution

Use the decreased total formula, plugging in 30 for the original amount and 0.40 for r since 40% must be expressed as a decimal:

$$\begin{aligned}\text{Decreased Total} &= (\text{Original Amount}) \times (1 - r) \rightarrow \\ \text{Decreased Total} &= (30) \times (1 - 0.40) = 30 \times 0.6 = 18\end{aligned}$$

Therefore, the discounted price of the toy is \$18.

Practice

What number is 60% more than 40?

The amount of monthly rainfall in a town in January was 10 inches, and in February it was 7 inches. By what percent did the monthly rainfall decrease?

A car originally sells for \$20,000, but is on sale for 10% off. How much money would a buyer save by purchasing the car on sale? (Do not include sales tax in your calculations.)

During the school year, Ashra would read 3 hours a week for fun. Over the summer, she would read 9 hours a week for fun. By what percent did her weekly leisure reading in the summer increase compared to what she did during the school year?

A book typically sells for \$30, but there is a 20% discount. After including a 7% sales tax, what is the sale price of the book?

Solutions

Use this formula, $\text{Increased Total} = (\text{Original Amount}) \times (1 + r)$, plugging in 40 as the original amount and 0.6 as r , since the percent must be expressed as a decimal:

$$\text{Increased Total} = (\text{Original Amount}) \times (1 + r) \rightarrow$$

$$\text{Increased Total} = (40) \times (1 + 0.6) = 40 \times 1.6 = 64$$

Use this formula, $\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change}$, plugging in 10 as the original amount and 7 as the new amount to solve for the percent it has changed:

$$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change} \rightarrow$$

$$\frac{7 - 10}{10} \times 100 = \frac{-3}{10} \times 100 = -30\%$$

Since this is negative, the monthly rainfall has decreased by 30%.

Rather than using the percent change formula, recognize that you simply need to calculate 10% of \$20,000 to find the amount saved:

$$20,000 \times 0.10 = 2,000$$

So, the amount of money saved is \$2,000.

Calculate the percent increase using this formula,
 $\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change}$, plugging in 3 for the original amount and 9 for the new amount:

$$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change} \rightarrow$$
$$\frac{9 - 3}{3} \times 100 = \frac{6}{3} \times 100 = 200\%$$

Since this is positive, there is a 200% increase in her weekly leisurely reading.

First, use this formula to find the discounted price:

$$\text{Decreased Total} = (\text{Original Amount}) \times (1 - r)$$

Use 30 as the original amount and r as 0.2 to represent the 20% discount:

$$\text{Decreased Total} = (30) \times (1 - 0.2) \rightarrow$$

$$\text{Decreased Total} = 30 \times 0.8 = 24$$

Now, use $\text{Increased Total} = (\text{Original Amount}) \times (1 + r)$ to find the price after adding the 7% sales tax, using 24 as the original amount and 0.07 as the r since 7% would be expressed as a decimal):

$$\text{Increased Total} = (\text{Original Amount}) \times (1 + r) \rightarrow$$

$$\text{Increased Total} = (24) \times (1 + 0.07) = 24 \times 1.07 = 25.68$$

So, the sale price after tax is added is \$25.68.

Simple Interest

The SAT will frequently test your skill in calculating how interest applied over time will affect the amount. First, *simple interest* enables you to calculate the interest over a set period of time.

Simple Interest

$$I = P \times r \times t$$

I = Amount of Interest
 P = Principal (Original Amount)
 r = Interest rate expressed as a decimal
 t = Time period the interest is applied (usually years)

➤ Example 1

Andrew has \$500 in his savings account, and it receives 1% interest over a year. How much interest would he have accumulated after the end of the year?

✓ Solution

Use the formula $I = P \times r \times t$ to solve for the amount of interest. The principal is \$500, the interest rate expressed as a decimal is 0.01, and the time period is one year.

$$\begin{aligned} I &= P \times r \times t \rightarrow \\ I &= 500 \times 0.01 \times 1 = 5 \end{aligned}$$

So, Andrew would earn \$5 in interest in one year.

➤ Example 2

Bridget's home price increases by 4% in simple interest (not compounded) over a three-year period. If her home at the beginning of the three-year period costs \$100,000, what is the price at the end of the period?

✓ Solution

Use $I = P \times r \times t$ to find the amount of interest over the three-year period. The principal is \$100,000, the rate is 0.04, and the time period is 3:

$$\begin{aligned} I &= P \times r \times t \rightarrow \\ I &= 100,00 \times 0.04 \times 3 = 12,000 \end{aligned}$$

Then, add 12,000 to the original price to find out the price of the house at the end of the period:

$$100,000 + 12,000 = 112,000$$

So, the price of her house after three years would be \$112,000.

Compound Interest

More frequently, the SAT Math will ask about *compound interest*:

Compound Interest

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

A = Future Value

P = Initial Value (Principal)

r = Interest Rate Expressed as a Decimal (r is positive if increasing, negative if decreasing)

t = Time

n = Number of Times Interest is Compounded over Time Period t

➤ Example 1

A savings bond has an initial value of \$500. The interest on the bond is 4%, compounded annually. What is the value of the savings bond after 2 years?

✓ Solution

Use the compound interest formula to solve. The initial value is \$500, the interest rate expressed as a decimal is 0.04, the time is 2 years, and the number of times it is compounded each year is 1.

$$\begin{aligned}
 A &= P \left(1 + \frac{r}{n} \right)^{nt} \rightarrow \\
 A &= 500 \left(1 + \frac{.04}{1} \right)^{(1 \times 2)} \rightarrow \\
 A &= 500 (1.04)^2 = 540.80
 \end{aligned}$$

So, the savings bond would have a value of \$540.80 after a 2-year time period.

▶ Example 2

The depth in a lake is 40 meters, and is decreasing by 1.2% each month, compounded. After 1 year, what will the depth of the lake be to the nearest hundredth of a meter?

✓ Solution

Use the compound interest formula, with 40 as P , -0.012 as r (since the depth is decreasing), n as 12 (since there are 12 months in a year), and t as 1 since there is 1 year.

$$\begin{aligned}
 A &= P \left(1 + \frac{r}{n} \right)^{nt} \rightarrow \\
 A &= 40 \left(1 + \frac{-0.012}{12} \right)^{12(1)} \rightarrow \\
 A &= 40 (0.999)^{12} \approx 39.52
 \end{aligned}$$

So, the depth of the lake after 1 year to the nearest hundredth of a meter is 39.52 meters.

SAT Math Strategy

In order to visualize percentage changes with variables, use 100 as an initial value.

▶ Example

The price, P , after n years for a television that initially costs x dollars is given in the function below.

$$P = xk^n$$

If the price of a television decreases by 10% each year, what is the value of the constant k ?

- (A) 0.1
- (B) 0.9
- (C) 10
- (D) 90

Solution

Most of the SAT Math problems you encounter will have concrete numbers in them. On the occasion that you have one with pure variables, it can often be difficult to visualize the potential changes. To make things easier, use 100 as an initial value—that way you can efficiently calculate the percent change from the initial value.

With this problem, let's then use 100 as the initial value for x . If the initial value decreases by 10%, it would be 90 the following year. So, we can plug in 90 for P , 100 for x , and 1 for n , then solve for k :

$$P = xk^n \rightarrow$$

$$90 = 100k^1 \rightarrow$$

$$90 = 100k \rightarrow$$

$$\frac{90}{100} = k \rightarrow$$

$$k = 0.9$$

So, the answer is **(B)**, 0.9 as the k value.

Practice Drill #1—Problem Solving and Data Analysis: Percentages

(If timing, take about 12 minutes to complete.)

What is 150% of 3,000?

- A) 1,500
- B) 2,000
- C) 4,500
- D) 5,500

At a certain university, 25% of graduating seniors earned an honors diploma. What is the number of students graduating with an honors diploma if there were a total of 480 graduating seniors at the university?

- A) 100
- B) 120
- C) 200
- D) 600

If a computer that regularly costs \$800 is on sale for 20% off, what would its sale price be (do not include sales tax in your calculation)?

- A) \$160
- B) \$200
- C) \$600
- D) \$640

Average Commute Time from Home to Work	Percentage of Respondents
Less than 10 minutes	25%
10–30 minutes	45%
Over 30 minutes	30%

If the number of respondents with an average commute of greater than 30 minutes is 240, how many people all together responded to this survey?

- A) 480
- B) 620
- C) 800
- D) 960

Sam purchases a total of 25 gift cards for his friends, with the gift cards being for retail stores or for restaurants. If he purchases 50% more retail store gift cards than restaurant gift cards, how many retail store gift cards did he purchase?

- A) 12.5
- B) 15
- C) 20
- D) 37.5

In a park, 15% of the parkgoers were children, 45% were male adults, and the remaining 8 were adult females. How many parkgoers were there all together?

- A) 20
- B) 24
- C) 30
- D) 36

Chelsea has a temperature of 98.6°F . The next day, she has a fever, with a temperature of 103.2°F . By what percentage, calculated to the nearest tenth of a percent, did her temperature increase from the first day to the second day?

- A) 4.5%
- B) 4.7%
- C) 5.1%

D) 5.4%

The U.S. stock market has returned a historical average of approximately 10.0% per year. If this rate of return were to continue into the future, what function would model the value, V , of an initial investment of V_0 in the stock market t years from now?

A) $V(t) = V_0 (10)^t$

B) $V(t) = V_0 (110)^t$

C) $V(t) = V_0 (1.1)^t$

D) $V(t) = V_0 (0.1)^t$

A new car is on sale for 15% off the sticker price. Maxwell is trading in his old car for a total of \$5,000—all of this counts as a credit toward the price paid for a new car purchase. Assuming there is no sales tax, if the car has a sticker price of x dollars, what amount will Maxwell have to pay, P , for a new car, accounting for the sale discount and the trade-in credit?

A) $P = 0.85x - 5,000$

B) $P = 0.85 + 5,000$

C) $P = 0.15x - 5,000$

D) $P = 0.15x + 5,000$

The gravitational force, F , is related to the gravitational constant, G , mass of one object, m_1 , and mass of another object, m_2 , and the distance between the centers of the objects, r , using the following equation:

$$F = \frac{Gm_1m_2}{r^2}$$

By what percent would the gravitational force between two objects change if the distance between them were doubled and the other quantities remained the same?

A) It would decrease by 50%.

- B) It would decrease by 75%.
- C) It would increase by 100%.
- D) It would increase by 200%.

Solutions

C) Take 150 and move the decimal point over two spots to the left. Then multiply this by 3,000:

$$1.5 \times 3,000 = 4,500$$

B) Calculate 25% of 480 by moving the decimal point two spots to the left in 25, then multiplying it by 480:

$$0.25 \times 480 = 120$$

D) Use the formula $\text{Decreased Total} = (\text{Original Amount}) \times (1 - r)$, with 800 is the original amount and r as 0.2:

$$\text{Decreased Total} = (\text{Original Amount}) \times (1 - r) \rightarrow$$

$$\text{Decreased Total} = 800 \times (1 - 0.2) = 800 \times 0.8 = 640$$

C) Use the formula $\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent}$ to calculate the total number of survey participants. Let's use x as the unknown number of total survey participants, 30 as the percent, and 240 as the “part”:

$$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent} \rightarrow$$

$$\frac{240}{x} \times 100 = 30 \rightarrow$$

Cross multiply $\rightarrow$

$$\frac{240}{30} \times 100 = x \rightarrow$$

$$8 \times 100 = x \rightarrow$$

$$800 = x$$

B) Set up an equation modeling the situation. The total number of gift cards is 25, and he purchased 50% more retail cards than restaurant cards. Use x as

he number of restaurant cards and use $1.5x$ as the number of retail cards, since it is 50% more:

$$x + 1.5x = 25 \rightarrow$$

$$2.5x = 25 \rightarrow$$

$$x = 10$$

Then, using the formula $\text{Number of Retail Gift Cards} = (\text{Original Amount}) \times (1 + r)$, calculate the number of retail gift cards purchased; 10 will be the original amount, and r will be 0.5:

$$\text{Number of Retail Gift Cards} = (\text{Original Amount}) \times (1 + r) \rightarrow$$

$$\text{Number of Retail Gift Cards} = 10 \times 1.5 = 15$$

Therefore, he will purchase 15 retail store gift cards. You could also try plugging in the answers to see which would fit the criteria.

A) The percent of female parkgoers will be what remains after subtracting the children and males:

$$100\% - 45\% - 15\% = 40\%$$

So, 40% of the parkgoers are female, and this number will be 8. Now, we can solve for the total number of parkgoers by using the equation $\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent}$; let's use x as the "whole," 40 as the "percent," and 8 as the "part."

$$\frac{\text{Part}}{\text{Whole}} \times 100 = \text{Percent} \rightarrow$$

$$\frac{8}{x} \times 100 = 40 \rightarrow$$

Cross Multiply $\rightarrow$

$$\frac{8}{40} \times 100 = x \rightarrow$$

$$0.2 \times 100 = x \rightarrow$$

$$20 = x$$

So, the total number of parkgoers is 20.

B) Use the formula $\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change}$ to calculate the percent change. The original value is 98.6 and the new value is 103.2:

$$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change} \rightarrow$$

$$\frac{103.2 - 98.6}{98.6} \times 100 \rightarrow$$

$$\frac{4.6}{98.6} \times 100 \approx 4.7\%$$

C) Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to model the future value of the investment. $V(t)$ will be A , P will be V_0 , r will be 0.1, n will be 1, and t will stay as a variable.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow$$

$$V(t) = V_0\left(1 + \frac{0.1}{1}\right)^{1(t)} \rightarrow$$

$$V(t) = V_0(1.1)^t$$

You could also solve this by looking at what makes the answer choices different and noticing that the only one that would show a 10% increase is choice **(C)**.

A) Take a 15% discount off the price of the new car to find what he will have to pay before the trade-in value is included:

$$\text{Decreased Total} = (\text{Original Amount}) \times (1 - r) \rightarrow$$

$$\text{Discounted Price} = x \times (1 - 0.15) = 0.85x$$

Then, subtract \$5,000 from the price paid to include the trade-in value:

$$P = 0.85x - 5,000$$

B) In doing the comparison, keep everything constant that remains constant, and change only what is different. The gravitational force, F , and the distance, r , are the only things that will change. Consider the original force:

$$F = \frac{Gm_1m_2}{r^2}$$

Now, double the distance, r , and see how the original force would change:

$$F = \frac{Gm_1m_2}{r^2} \rightarrow$$

$$\text{New } F = \frac{Gm_1m_2}{(2r)^2} = \frac{Gm_1m_2}{4r^2} = \frac{1}{4} \left(\frac{Gm_1m_2}{r^2} \right) = \frac{1}{4}F$$

So, the new force is $\frac{1}{4}$ of the original value, which means it has decreased by $\frac{3}{4}$ its original value. Convert $\frac{3}{4}$ to a percentage by using a proportion:

$$\frac{3}{4} = \frac{x}{100} \rightarrow$$

$$0.75 = \frac{x}{100} \rightarrow$$

$$75\% = x$$

Therefore, the gravitational force between the two objects would decrease by 75% if the distance between them were doubled.

Practice Drill #2—Problem Solving and Data Analysis: Percentages

(If timing, take about 12 minutes to complete.)

Fill-In Practice: Write your answer in the underlined blank under each question.

What number is 130% of 50?

Answer: _____

In the United States in 2019, approximately 62% of utility electricity came from fossil fuels, 20% from nuclear, and 18% from renewable sources. What fraction of U.S. utility electricity in 2019 came from nuclear?

Answer: _____

A wind turbine has a maximum electrical generating capacity of 200 kilowatts per hour, and it operates on average at 25% of this maximum capacity. If the turbine operates at its average capacity for an entire day, how many kilowatts of electricity would it produce?

Answer: _____

Mascot Preference	Percent of Students Who Prefer Mascot
Tiger	10%
Lion	15%
Bear	30%
Eagle	25%
Dog	20%

A college had all of its students vote on their preference for a new school mascot. If the number of students who preferred Bear exceeded the number who preferred Dog by 240 students, how many students selected Eagle?

Answer: _____

A toy store sells only dolls and games; 70% of the toys it sells are dolls and 30% are games. The store sells a total of 400 toys. How many more dolls than games does the store sell?

Answer: _____

Multiple-Choice Practice

A house was initially listed for sale at \$200,000. After three months, the sellers still had not sold it and decided to drop the price by 15%. What would the new price be?

- A) \$170,000
- B) \$185,000
- C) \$215,000
- D) \$230,000

A school district is doing a ten-year projection of its student enrollment. If at the start of the ten years there are 2,000 students in the district, and the student enrollment increases by 5% each year, what would be the enrollment t years from now?

- A) $2,000 (1.5)^t$
- B) $2,000 (1.05)^t$
- C) $2,000 (.05)^t$
- D) $2,000 (5)^t$

In the year 2010, New York City had a population density of 27,016 people per square mile, and San Francisco had a population density of 17,246 people per square mile. The population density in San Francisco was what percent lower than the population density in New York in 2010, calculated to the nearest tenth of a percent?

- A) 36.2%
- B) 41.6%
- C) 53.4%
- D) 63.8%

The elimination half-life of a medicine is the amount of time it takes for the concentration of the medicine in the body to be reduced by half. If a particular medicine starts at a dose of 100 mg that is fully absorbed in the body and has an elimination half-life of 8 hours, how much of the medicine remains in the body after 1 day?

- A) 6.25 mg
- B) 12.5 mg
- C) 25 mg
- D) 36 mg

One serving of a cereal is $\frac{2}{3}$ of a cup and has 20% of the daily recommended amount of only fiber, which is 6 grams. If another cereal has a serving size of

cup, how many grams of fiber should it have to provide 30% of the daily value of fiber?

- A) 4 grams
- B) 6 grams
- C) 9 grams
- D) 12 grams

Solutions

65) Move the decimal point in 130 to the left two spots, then multiply by 50:

$$1.3 \times 50 = 65$$

0.2 or $\frac{1}{5}$) Convert the 20% from nuclear to a percentage by reducing the fraction to its simplest form:

$$\frac{20}{100} \rightarrow \frac{1}{5}$$

This could also be expressed as a decimal, 0.2.

1,200) Multiply the number of kilowatts per hour, 200, by 0.25 to get the 25% operational level. Then, multiply this by 24 since there are 24 hours in a day:

$$200 \times 0.25 \times 24 = 1,200$$

So, it would produce 1,200 kilowatts of electricity in a day.

600) The Bear students are 30% of the total, and the Dog students are 20% of the total. The difference between them is 10%, so 10% of the total is 240. Multiply 240 by 10 to get the total number of students (since 10% multiplied by 10 is 100%):

$$240 \times 10 = 2,400$$

Then, take 25% of the total:

$$0.25 \times 2,400 = 600$$

160) Calculate the number of dolls sold (70% of 400):

$$0.7 \times 400 = 280$$

Then, calculate the number of games sold (30% of 400):

$$0.3 \times 400 = 120$$

Then, subtract 120 from 280 to find how many more dolls are sold than games:

$$280 - 120 = 160$$

Alternatively, if you notice the difference between them must equal 40% of the total toys sold, you could just take 40% of 400 and get the same result of 160.

A) Calculate the discounted price by using this formula,

Decreased Total = (Original Amount) $\times$ (1 - r), with 200,000 as the original amount and r as 0.15:

$$\text{Decreased Total} = (\text{Original Amount}) \times (1 - r) \rightarrow$$

$$\text{Sale Price} = 200,000 \times (1 - 0.15) = 200,000 \times 0.85 = 170,000$$

B) Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to determine what the enrollment would be. A represents the enrollment, P is the original number of 2,000 students, r is 0.05 (5% expressed as a decimal), n is 1 since the compounding is annual, and t will remain a variable:

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow$$

$$\text{Enrollment} = 2,000\left(1 + \frac{0.05}{1}\right)^{(1)t} \rightarrow$$

$$\text{Enrollment} = 2,000(1.05)^t$$

A) Use the formula $\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change}$ to calculate the percent difference between the population density of New York City and San Francisco. The “original” value will be 27,016 and the “new” will be 17,246:

$$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100 = \text{Percent Change} \rightarrow$$

$$\frac{17,246 - 27,016}{27,016} \times 100 \rightarrow$$

$$\frac{-9,770}{27,016} \times 100 \approx -36.2\%$$

Since this is negative, the population density in San Francisco is about 36.2% lower than it is in New York City.

B) During a one-day period of 24 hours, the medicine will have gone through three half-life cycles, since each half-life cycle is 8 hours and 8 goes into 24 three times. So, to find out how much of the medicine is left after three half-life cycles, take 50% of the original amount of 100 three times:

$$100 \times 0.5 \times 0.5 \times 0.5 = 12.5$$

C) Sort out the essential information here, as there is irrelevant information given. The serving sizes of the cereal are not important—what is important is providing the correct amount of fiber. The first cereal provides 20% of the daily value of fiber in its serving; the second cereal provides 30% of the daily value of fiber in its serving. Set up a proportion to solve for x , the number of grams of fiber in the second cereal:

$$\frac{30}{20} = \frac{x}{6} \rightarrow$$

$$1.5 = \frac{x}{6} \rightarrow$$

$$6 \times 1.5 = 9 = x$$

Problem Solving and Data Analysis—Surveys

Surveys are a part of everyday life, and the SAT will assess your skill in interpreting surveys. One of the biggest things emphasized will be determining whether a sampling method is appropriate.

Sampling Methods

In general, to get the best results in a survey, have the sample be as large and as random as possible. The larger and more random the survey, the less the potential bias will be in the results.

Let's first look at some ways a survey could be POORLY conducted.

- **Sample is chosen non-randomly:**

- Example: Researcher interviews only parents and caregivers at a playground on their thoughts about increased funding for extracurricular activities at schools to determine what all the citizens in the community think about increased extracurricular funding. *Why is this problematic?* If the researcher is interested in sampling the community as a whole, by limiting the respondents to parents and caregivers who have congregated at a playground, the researcher will oversample those who are already interested in children's activities and/or have children.
- Example: A city worker samples city residents who have memberships to the city's community center about whether they would support a tax levy for an additional city community center. *Why is this problematic?* If the respondents are all members of the city's community center, they are using this service already. So, it is more likely they will support increased funding for a city service they enjoy. The worker should instead include respondents who are not already members of the community center.

- **Participants can volunteer:**

- Example: A politician has a survey on her website asking whether she or her opponent won a political debate. *Why is this problematic?* Supporters are more likely to be on her website and believe their preferred candidate—i.e., the politician conducting the survey—won the debate. This will result in a sample of respondents skewed toward those who already supported the candidate.
- Example: A theme park worker asks people who are leaving the theme park to complete a 10-minute customer survey about their experience. *Why is this problematic?* This will result in a sample skewed toward those who have the time to complete the survey and

feel passionately enough about their experience (for good or for bad) to want to answer questions.

- **Sample size is too small:**

- Example: A student, who lives in a city of 100,000 people, surveys four city residents about their favorite restaurant in the city and draws conclusions about the city's favorite restaurant. *Why is this problematic?* Four people is too small of a sample when the population of the city is 100,000.

Now, let's consider some ways a survey could be conducted WELL.

- **Responses are randomly collected.**

- Example: To survey a group of 1,000 people, a random number generator selects which of the 1,000 people will be respondents.
- Example: To select 100 respondents for a political survey in a state, a researcher randomly selects phone numbers from a database of all mobile phones and landlines in the state.

- **The sample size is sufficiently large.**

- Example: To predict the winner of an upcoming senatorial election, a random sample of 1,000 likely voters is conducted.

- **All groups in a survey are fairly represented.**

- Example: To test the effectiveness of a medicine in the population as a whole, a university ensures that there are sufficient patients in the study from different genders, ages, and ethnicities.
- Example: To determine the popularity of a shift in the school schedule among staff members, a school superintendent ensures that adequate representation from all types of school district workers is given—teachers, administrators, custodians, bus drivers, etc.

Use *margin of error* to quantify the quality of a survey's conclusions.

Margin of error is the maximum expected difference between the actual parameter and the sample parameter. If a survey is conducted poorly, it will have a HIGH margin of error. If it is conducted well, it will have a LOW margin of error.

➤ **Example**

If a pollster conducts a poll that says 40% of the voters will vote in favor of Candidate 1, with a 3% margin of error, which of the following could be the actual value of the percent of votes cast in the election for Candidate 1?

- (A) 35%
- (B) 38%
- (C) 44%
- (D) 47%

✓ Solution

Since the margin of error is 3%, that means that the actual results are projected to be between $\pm 3\%$ of 40%, so between 37% and 43%. So, the correct answer is choice **(B)**, since 38% falls within this range.

Drawing Conclusions

SAT Math Strategy

Limit generalizations about a survey to what is directly supported by the results.

› Example 1

A researcher asks 100 randomly selected survey participants in a city their favorite place for outdoor exercise. The survey has a low response rate, and the researcher concludes that the residents are not exercising sufficiently and that a “Get Outdoors Health Campaign” is needed to motivate city residents to exercise outdoors. Is this conclusion warranted based on the given information?

✓ Solution

No, it is not supported by the given information since there are many potential reasons why the response rate to the survey could be low:

- Perhaps the deadline to return the survey was too short.
- Perhaps the surveys did not reach the participants.
- Perhaps the potential respondents did not want to participate in research.

Moreover, the survey did not ask participants if they exercise outside—instead, it only asked their favorite location to do so. Given what the survey asked, a conclusion about whether or not city residents are exercising outside and need to be motivated with an advertising campaign cannot be drawn.

► Example 2

A city worker wants to determine the percent of the city's residents that are fluent in both Spanish and English. The worker writes the survey in Spanish and mails it to 400 randomly selected addresses in the city. Thirty of the surveys are returned, and 90% of them state that their household members are fluent in both Spanish and English. Would it be correct to generalize that 90% of the town's residents are fluent in both Spanish and English?

✓ Solution

No, because of the survey design. If household members were not able to read Spanish, they would not be able to understand the survey and could not respond. Therefore, households in which the residents were only fluent in English would be far less likely to be included in the results. Households that were able to read Spanish would understand the instructions and would be much more likely to respond, skewing the results to show a higher proportion of bilingual households.

Practice Drill—Problem Solving and Data Analysis: Surveys

(If timing, take about 12 minutes to complete.)

To find out which world language they wanted to study in high school, 250 junior high students were surveyed. Of the students surveyed, 75 indicated

hey wanted to study Chinese. Which conclusion is best supported by this data?

- A) Approximately 30% of the junior high students surveyed were interested in studying Chinese.
- B) Approximately 75% of the junior high students surveyed were interested in studying Chinese.
- C) Approximately 25% of all students in the town were interested in studying Chinese.
- D) Approximately 35% of all students in the town were interested in studying Chinese.

According to a recent student survey, 40% of high school seniors in the United States have taken a college credit class while in high school. If there are 3.7 million students in this year's high school graduating class in the United States, approximately how many of these graduating students likely took a college credit class while in high school?

- A) 148,000
- B) 222,000
- C) 1,480,000
- D) 2,220,000

For an upcoming election, a pollster estimates that 55% of voters will vote for Ballot Issue #1, given a margin of error of $\pm 3\%$. If there are 200,000 voters in the upcoming election, what would the pollster estimate to be the number, x , who will vote for Ballot Issue #1?

- A) $90,000 < x < 96,000$
- B) $104,000 < x < 116,000$
- C) $120,000 < x < 124,000$
- D) $224,000 < x < 248,000$

A newspaper surveyed 2,000 likely voters about their preference for a candidate in the upcoming election. The results are compiled in the table below:

Candidate	Vote Total
Candidate A	980
Candidate B	764
Candidate C	155
No Preference	101

Based on these results, which of the following would be a true statement?

- A) 49% of the survey respondents preferred candidate A.
- B) 98% of the survey respondents preferred candidate A.
- C) 16% of survey respondents preferred candidate C.
- D) 46% of survey respondents preferred candidate C.

A randomly selected survey of 2,000 dentists finds that 600 prefer Brand A dental floss. Based on this information, out of a group of 1,000 randomly selected dentists, how many will likely prefer Brand A dental floss?

- A) 200
- B) 300
- C) 800
- D) 900

A political scientist conducts a survey about approval for a state's governor. The results of the random sample of registered voters in the state are given in the table below:

% Who Approve	% Who Disapprove	Margin of Error
55	45	5

What action by the political scientist would most likely decrease the survey's margin of error?

- A) Conduct an identical survey in a different state
- B) Conduct an identical survey in a different country

- C) Decrease the survey sample size
- D) Increase the survey sample size

A travel agency wanted to determine which types of vacation destinations would be most popular for potential clients. They randomly sampled 100 adults within the boundaries of the travel agency's city and found that the majority of the respondents preferred a beach destination. What is the largest group for which these results of the market survey could accurately be generalized?

- A) The child residents of the travel agency's city
- B) The adult residents of the travel agency's city
- C) All residents of the travel agency's city
- D) All citizens of the travel agency's country

A medical researcher surveyed a random sample of nursing home residents and estimated between 46% and 54% of the residents had a family member who lived within 25 miles. What is another way she could report her estimate?

- A) 25% with an 8% margin of error
- B) 46% with an 8% margin of error
- C) 50% with a 4% margin of error
- D) 54% with a 4% margin of error

At an arboretum, a botanist found that out of 100 randomly selected trees from the arboretum, 30 were evergreen and 70 were deciduous. If there were a total of 1,000 trees in the arboretum, which inequality expresses how many of the remaining trees, E , would have to be evergreen if more than half of all the trees in the arboretum were evergreen?

- A) $E > 470$
- B) $E < 470$
- C) $E > 590$

D) $E < 590$

One hundred randomly selected users of a television streaming service were surveyed as to their preferred use for the service. Of these, 35% preferred to ‘binge watch’ one show until they had seen all the episodes for that show, and 65% preferred to skip around to watch different shows. What is the most accurate conclusion based on these results?

- A) It is more likely that a randomly selected user of the service prefers binge watching to show skipping.
- B) It is more likely that a randomly selected user of the service prefers show skipping to binge watching.
- C) Exactly 65% of all users of the streaming service prefer binge watching over show skipping.
- D) Exactly 65% of all users of the streaming service prefer show skipping over binge watching.

Solutions

A) We can eliminate choices (C) and (D) since the survey is limited to students at the junior high and would not be representative of all students in the town. The correct answer is choice **(A)** because 75 of the 250 junior high students said they wanted to study Chinese—this equates to 30%.

$$\frac{75}{250} \times 100 = 30\%$$

C) Based on the survey data, 40% of high school seniors in the United States have taken a college credit class. Therefore, if we take 40% of the total number of students in the United States who are graduating from high school, we will get the approximate number of the graduating students who likely took a college credit class:

$$0.40 \times 3,700,000 = 1,480,000$$

B) Since the margin of error is 3%, the likely number of voters will be between 52% and 58% of the 200,000 total.

$$0.52 \times 200,000 = 104,000$$

$$0.58 \times 200,000 = 116,000$$

So, the estimated number of voters who will vote for Ballot Issue #1 is 104,000 < x < 116,000.

A) There are 2,000 total respondents to the survey, so determine what percent of this total Candidate A would be:

$$\frac{980}{2,000} \times 100 = 49\%$$

Thus, 49% of survey respondents preferred Candidate A. The other options are based on incorrect calculations with the vote totals in the table.

B) Set up a proportion to solve for the number of dentists out of the 1,000 randomly selected ones who will prefer Brand A:

$$\frac{600}{2,000} = \frac{x}{1,000} \rightarrow$$

Cross multiply $\rightarrow$

$$\begin{aligned} \frac{600}{2,000} \times 1,000 &= x \rightarrow \\ 300 &= x \end{aligned}$$

D) To decrease the margin of error, make the survey sample as large and as random as possible. Choices (A) and (B) are incorrect because conducting the same survey in a different location will not better inform us about the approval within the state; they would be conducting the studies outside the considered population. Choice **(D)** is correct because increasing the sample size will make the survey data set larger, therefore decreasing the margin of error.

B) The travel agency did a random sample of adults within the boundaries of the travel agency's city, so the generalization will be for adult residents of that city. It will not be choice (A) since children were not surveyed. It will not be choice (C) because that would include both children and adults in the generalization. And it will not be choice (D) because we do not know the specifics of other areas in the country and how they would respond.

C) The number 50 is halfway between 46 and 54, with 4 units separating 46 from 50 and 4 units separating 54 from 50. So, the researcher could report the estimate as 50% with a 4% margin of error since this would encompass values between 46% and 54%.

A) One hundred trees have already been selected, so there would be $1,000 - 100 = 900$ trees remaining. Half of all the trees in the arboretum would be 500. In the survey sample, there were 30 evergreen trees. So, there must be enough evergreen trees in the remaining 900 to give a total of over 500. Solve this inequality to reach the solution:

$$\begin{aligned} E &> 500 - 30 \rightarrow \\ E &> 470 \end{aligned}$$

B) Choices (C) and (D) assert too much precision by stating that “exactly” 55% of the users must have a particular preference. Since 65% of the survey respondents said they like to skip around to different shows, and only 35% of the respondents preferred to binge watch, it would be reasonable to conclude that a randomly selected user of the service prefers show skipping to binge watching.

Problem Solving and Data Analysis—Graphs and Data Interpretation

Probability

Probability is defined as “the likelihood that a given event will happen,” expressed as a decimal or fraction.

- If the probability that it will rain tomorrow is 0.5, there is a 50% chance it will rain.
- If the probability that it will snow tomorrow is 0, there is a 0% chance it will snow.
- If the probability that it will be cloudy tomorrow is 1, there is a 100% chance it will be cloudy.

To calculate probability, take the number of ways an event can happen and divide it by the total number of possible outcomes:

$$\frac{\text{Number of Ways Event Can Happen}}{\text{Total Number of Possible Outcomes}} = \text{Probability of Event}$$

➤ Example

If I have a total of 30 books on my bookshelf and 10 of them are novels, what is the probability that a randomly selected book from my bookshelf would be a novel?

✓ Solution

The “number of ways an event can happen” is 10 and the “total number of possible outcomes” is 30, so divide 10 by 30 to determine the probability:

$$\frac{10}{30} = \frac{1}{3}$$

So, the probability that a randomly selected book from the bookshelf would be a novel is $\frac{1}{3}$.

Practice

On a multiple-choice test, there are four choices per question: (A), (B), (C), and (D). What is the probability that on a given question a test taker who is randomly selecting his answer will select choice (B)?

In John’s wallet, he has five one-dollar bills, two five-dollar bills, and three ten-dollar bills. If a bill is selected at random from his wallet, what is the possibility it will be a five-dollar bill?

An advertiser finds that for 200 randomly selected online ad placements, six users will click through to the business website. Based on this data, what is the probability that a randomly selected ad placement will result in the user clicking through to the business website?

Solutions

There are a total of four possible events, and choice (B) is one of the potential outcomes. So, divide 1 by 4 to find the probability: $\frac{1}{4}$.

John has a total of $5 + 2 + 3 = 10$ bills in his wallet. Out of these ten bills, two are five-dollar bills. So, divide 2 by 10 to find the probability:

$$\frac{2}{10} = \frac{1}{5}$$

There are 6 possible events out of 200 potential outcomes, so divide 6 by 200 to determine the probability:

$$\frac{6}{200} = 0.03$$

So, the probability that a randomly selected ad placement will result in the user clicking through to the business website is 0.03.

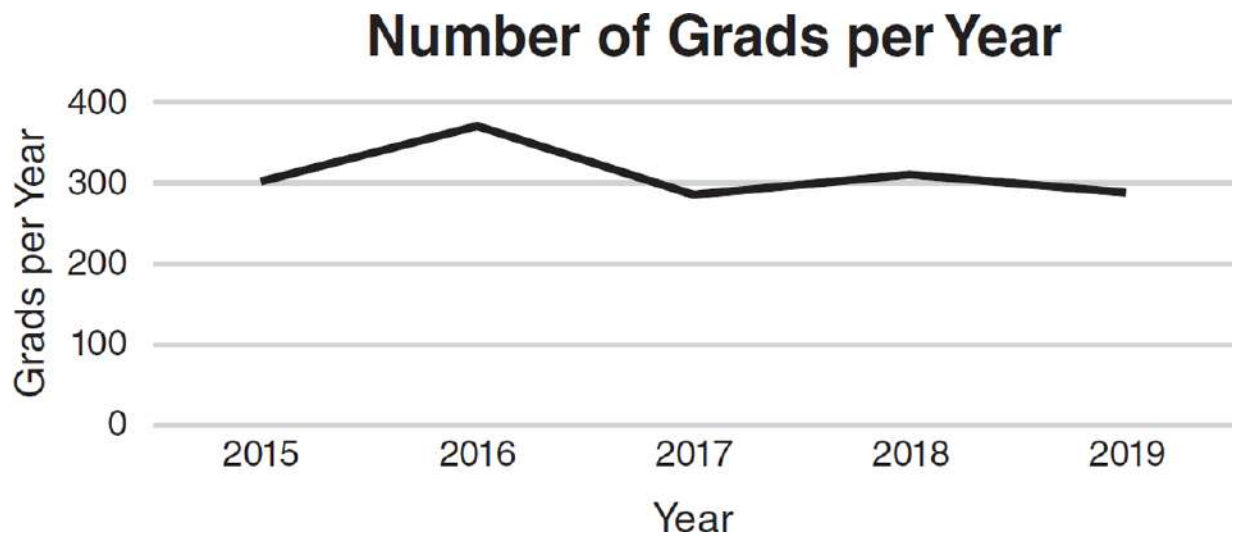
Graph Interpretation

The SAT will assess your skill in interpreting a variety of graphs—you will be asked to find maximums, minimums, overall trends, and more.

SAT Math Strategy

Carefully examine the units on the axes to avoid careless errors.

► Example

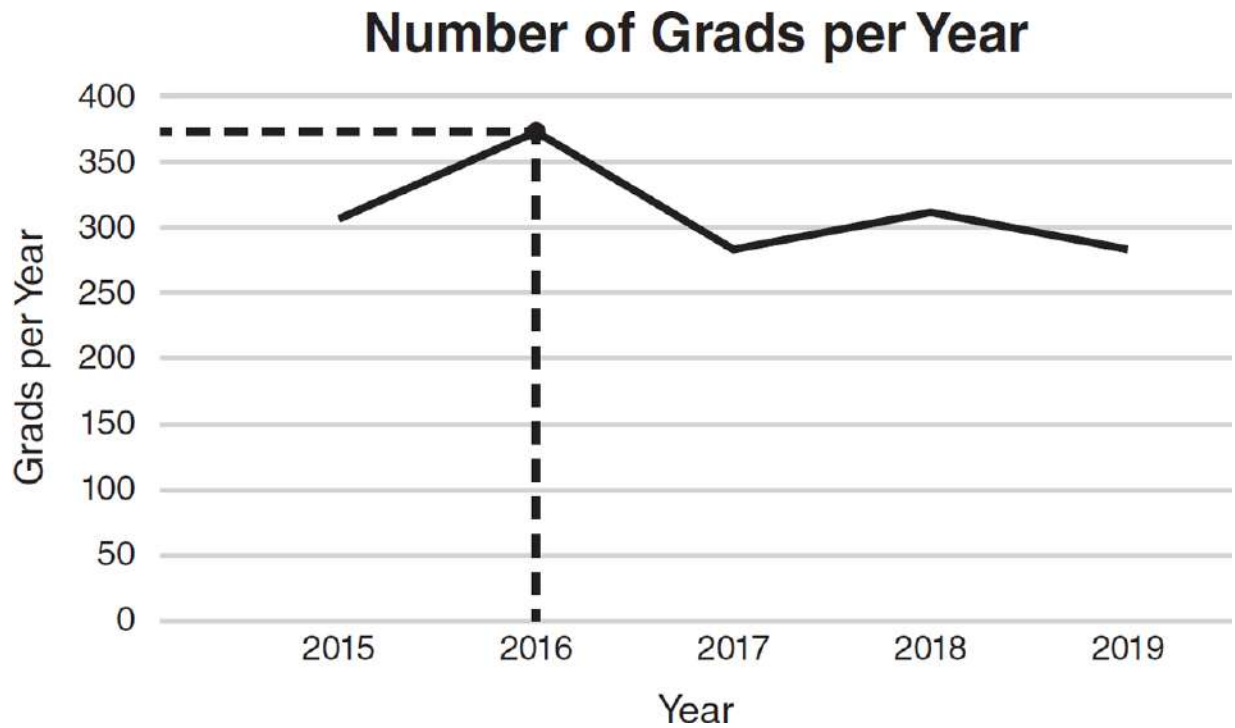


The number of graduates per year from Jackson High School are given in the line graph above.

During what year of this time period was the number of graduates the greatest?

Solution

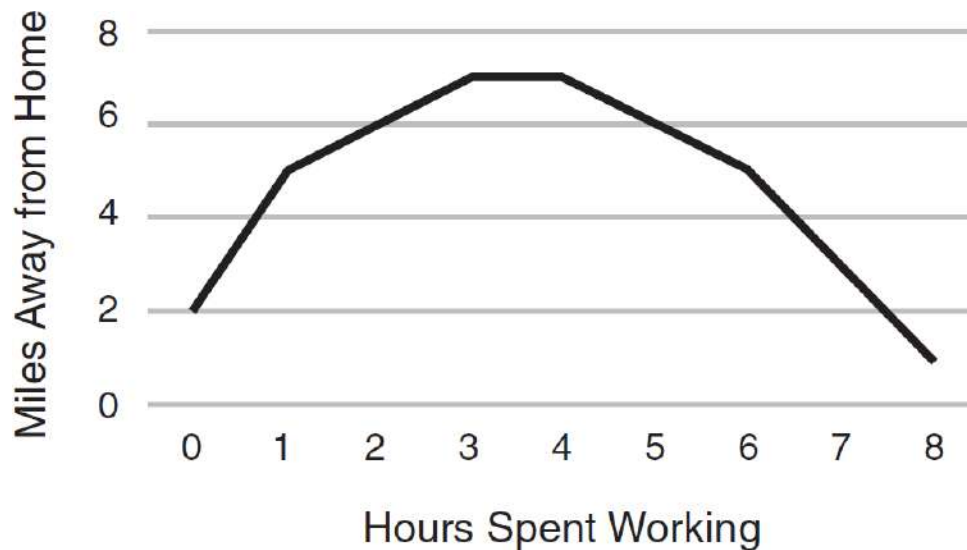
Examine the graph to see that the x -axis has the “years” and the y -axis has “grads per year.” So, we need to find where the “grads per year” are at their highest value and the corresponding year.



Drawing on the graph will help you visualize that the greatest number of grads per year is approximately 370, and that this occurs in year 2016. So, the answer is 2016.

Practice

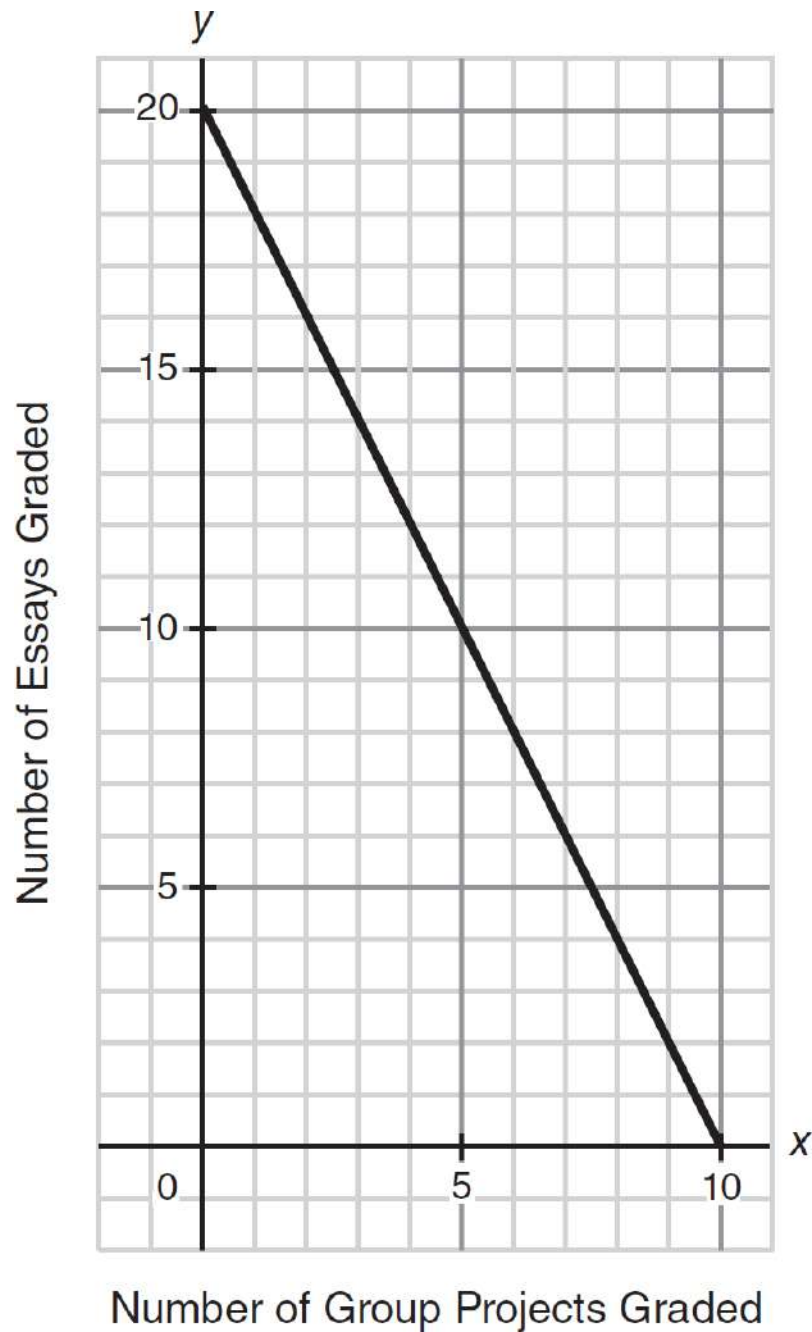
A delivery driver plots how far he is from his home throughout his workday.



How far from home is the worker when he starts his workday?

For approximately how long is the driver at least 5 miles from home?

A teacher has a total of 100 minutes he can spend grading essays and group projects. The various ways he could allocate his time are graphed below:



If the teacher spends the full 100 minutes grading projects and essays, and he grades 5 group projects, how many essays will he grade within this time?

How long does it take for him to grade an essay?

Solutions

2 miles. Zero hours of working would correspond to the beginning of the driver's workday. At an x value of 0, the y value is 2 for the number of miles away from home. So, he will be 2 miles from home when he starts his workday.

5 hours. From 1 hour spent working to 6 hours spent working, the miles away from home is at least 5. So, there will be a total of $6 - 1 = 5$ hours he is at least 5 miles from home.

10 essays. Examine the line and find that when there are 5 group projects graded, the y -value representing the number of essays graded is 10. So, there will be 10 essays graded in this time.

5 minutes. To find this, look at the situation when the teacher does nothing but grade essays. This is at the point $(0, 20)$ —no group projects are graded, and 20 essays would be graded. Since the teacher is spending a total of 100 minutes grading, find the number of minutes it takes to grade a single essay by dividing 100 minutes by 20 essays:

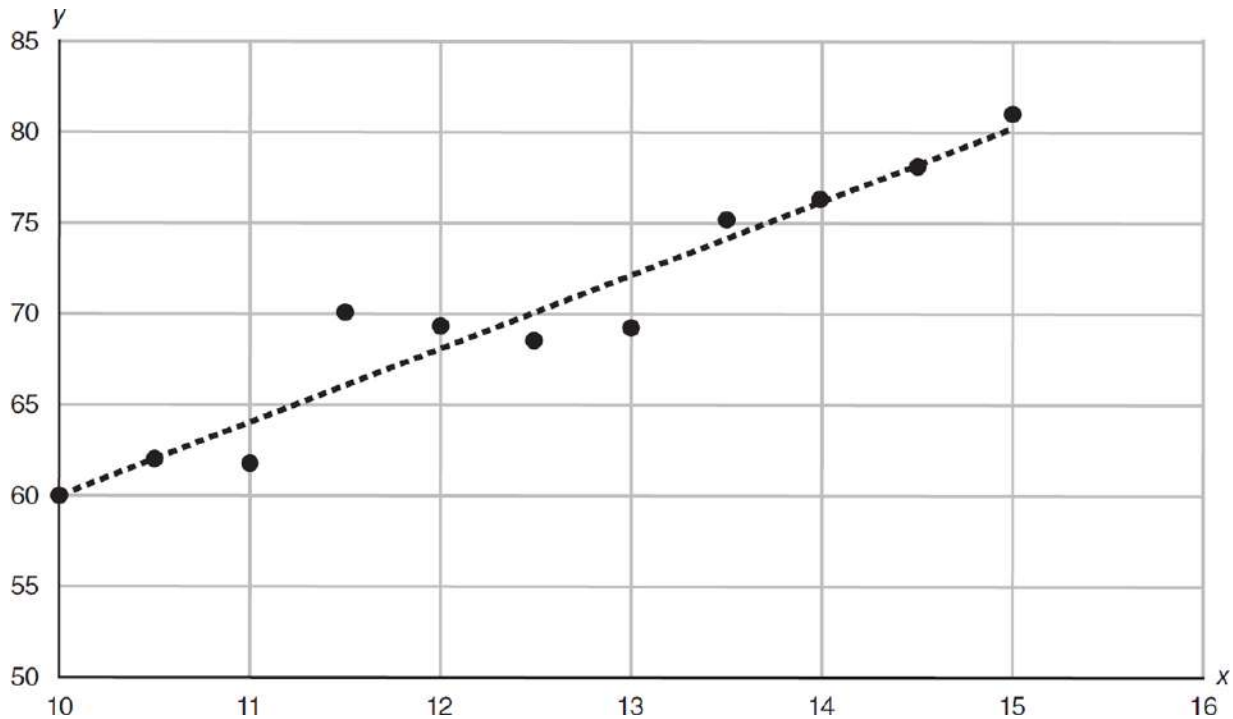
$$\frac{100}{20} = 5$$

Scatterplots

Scatterplots provide a graph of different points that together will show a relationship among data. To see the relationship among the data, visualize a **line of best fit** that shows a line that would best approximate the data points. (On the SAT, the edge of your scrap paper can give you a great way to approximate a straight line.) Here is an example of a scatterplot with a line of best fit:

► Examples

Consider the line of best fit for the scatterplot graphed below.



What is the equation for the line of best fit, with the slope and y -intercept rounded to the nearest whole integer values?

✓ Solution

Be careful to examine the x - and y -coordinates in the graph—they do not begin at $(0, 0)$. First, pick two points for which it will be easy to find the slope: $(10, 60)$ and $(15, 80)$ will work.

$$\frac{\text{Change in Y}}{\text{Change in X}} = \frac{80 - 60}{15 - 10} = \frac{20}{5} = 4$$

So, the slope of the line is 4. Now, plug in a point that is in the line of best fit, such as $(10, 60)$, to solve for the y -intercept:

$$y = mx + b \rightarrow$$

Plug in 4 for m , the slope:

$$y = 4x + b \rightarrow$$

Plug in (10, 60) as a point to solve for b :

$$60 = 4(10) + b \rightarrow$$

$$60 = 40 + b \rightarrow$$

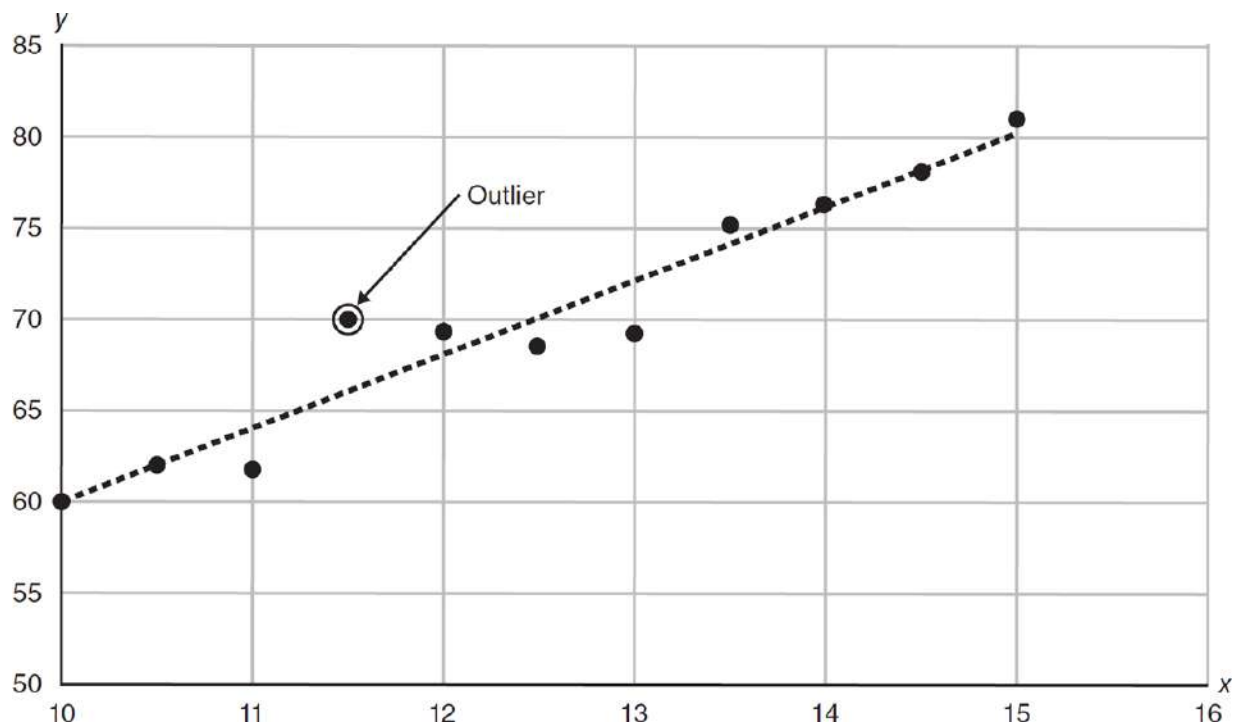
$$20 = b$$

So, the equation for the line of best fit is $y = 4x + 20$ since the line has a slope of 4 and a y -intercept of 20.

2. Which point is an “outlier”—an observation that lies farthest from the values predicted by the line of best fit?

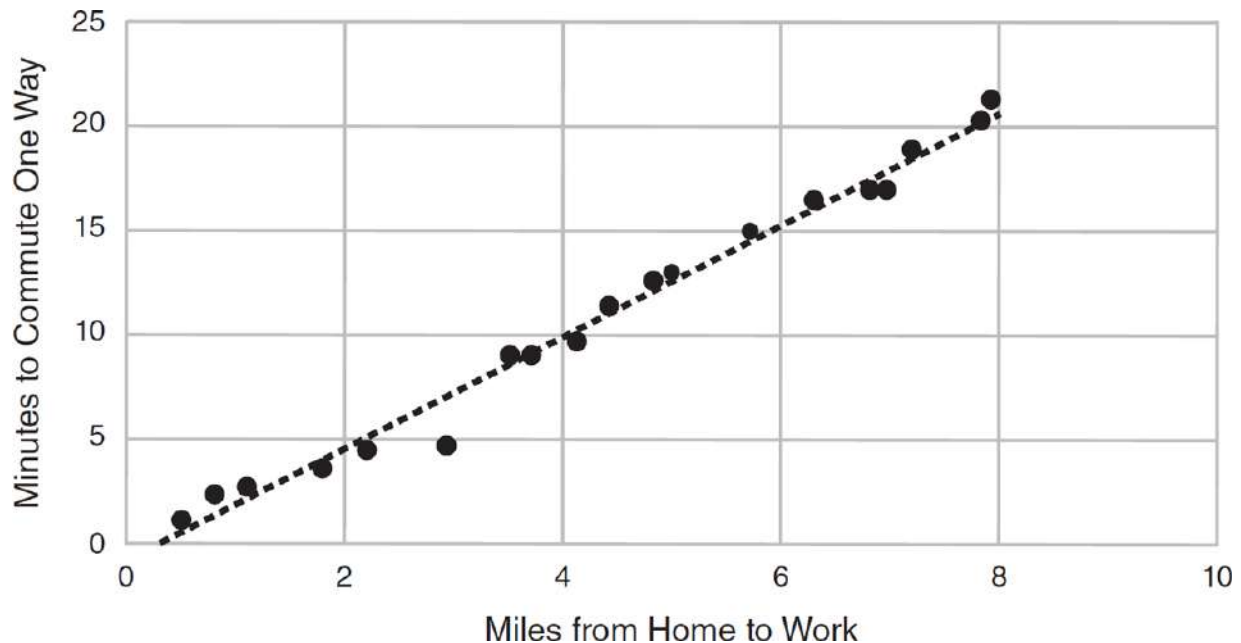
✓ Solution

Find the point that is most distant from the line of best fit:



(11.5, 70) is about 4 units above the predicted value for the line of best fit. So, this would be an outlier in the graph.

Practice



Twenty commuters recorded the distance from their homes to work and the time of their commutes on a particular day.

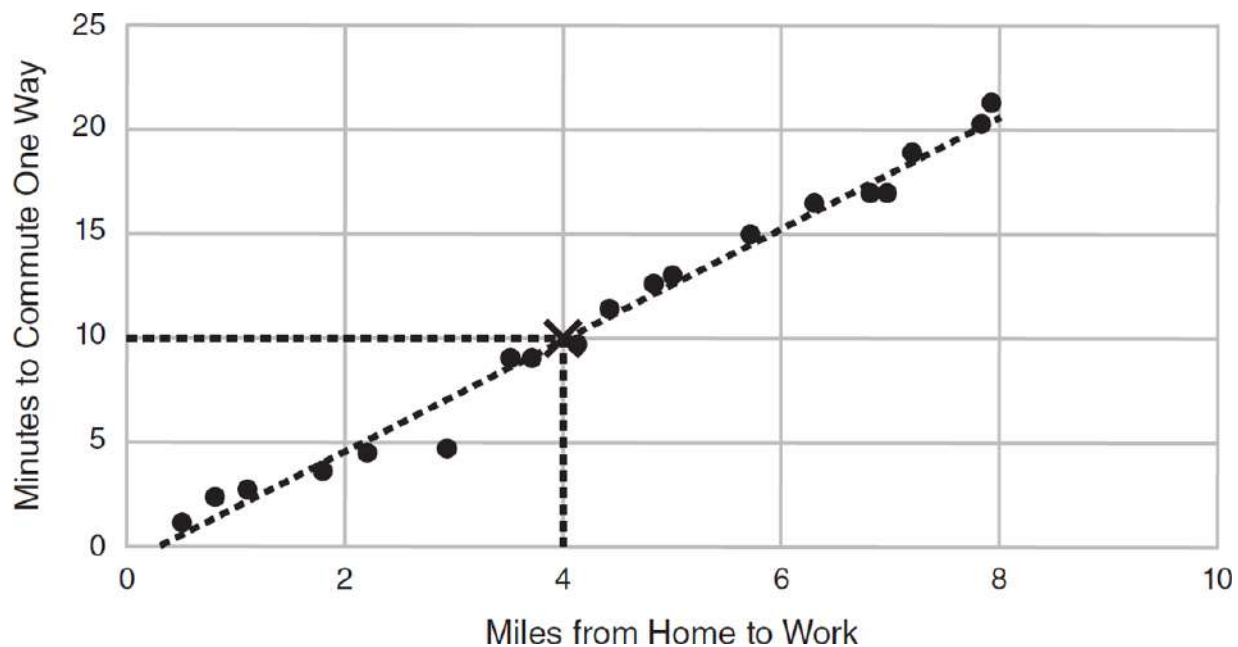
Based on the line of best fit, if someone lived 4 miles from work, how long would their commute time most likely be?

What value on the graph is most different from what would be predicted based on the trend?

For how many of the commuters was their actual commute time shorter than what would be predicted?

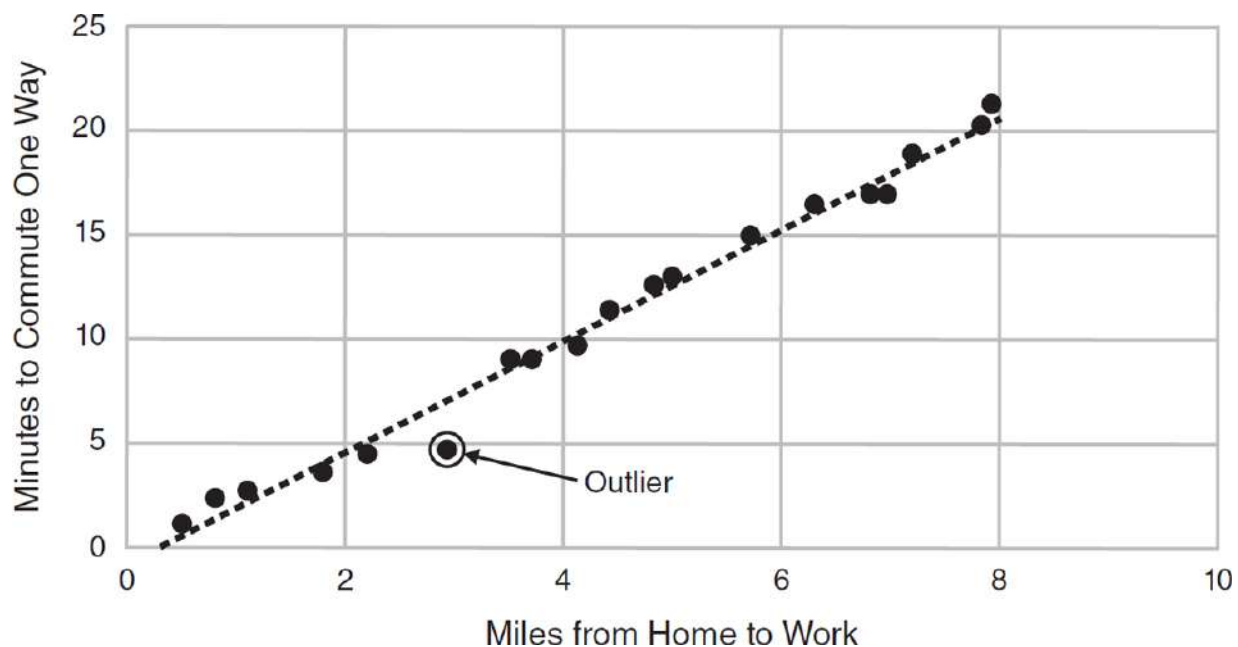
Solutions

1.



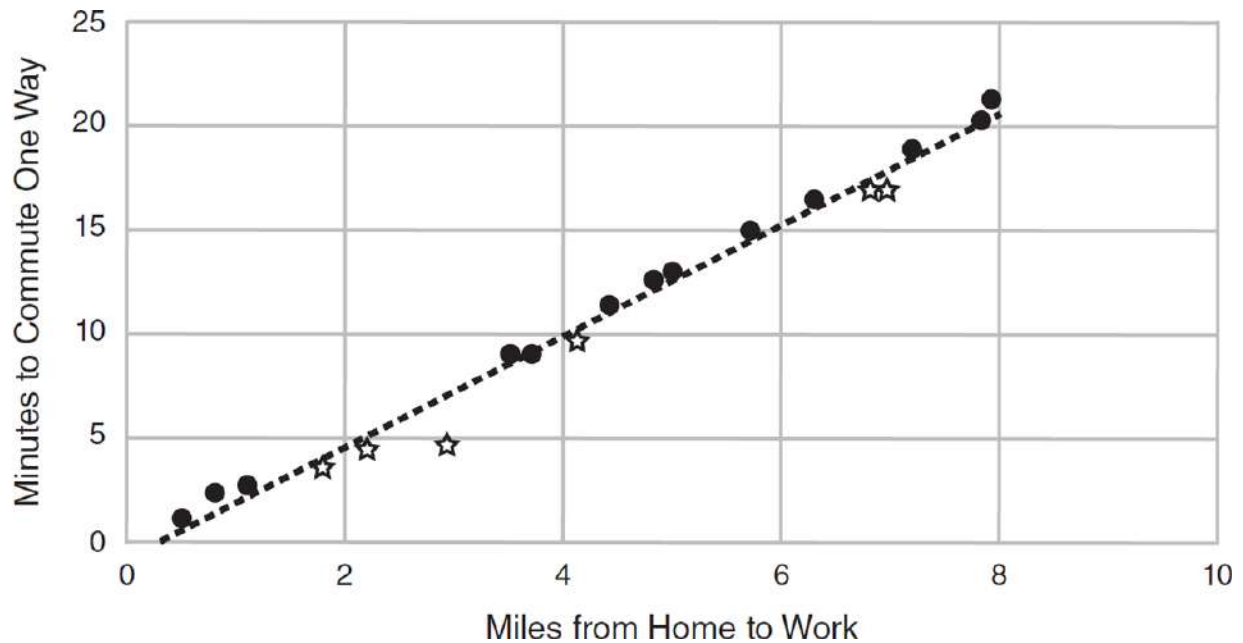
At 4 miles from home to work, the predicted number of commuting minutes is approximately 10, as you can see in the graph above.

2.



The value that is most different from what is predicted is the point (2.9, 4.6)—this point is an “outlier” since it is far away from the line of best fit.

3. Mark and count the number of points that fall *underneath* the line of best fit:

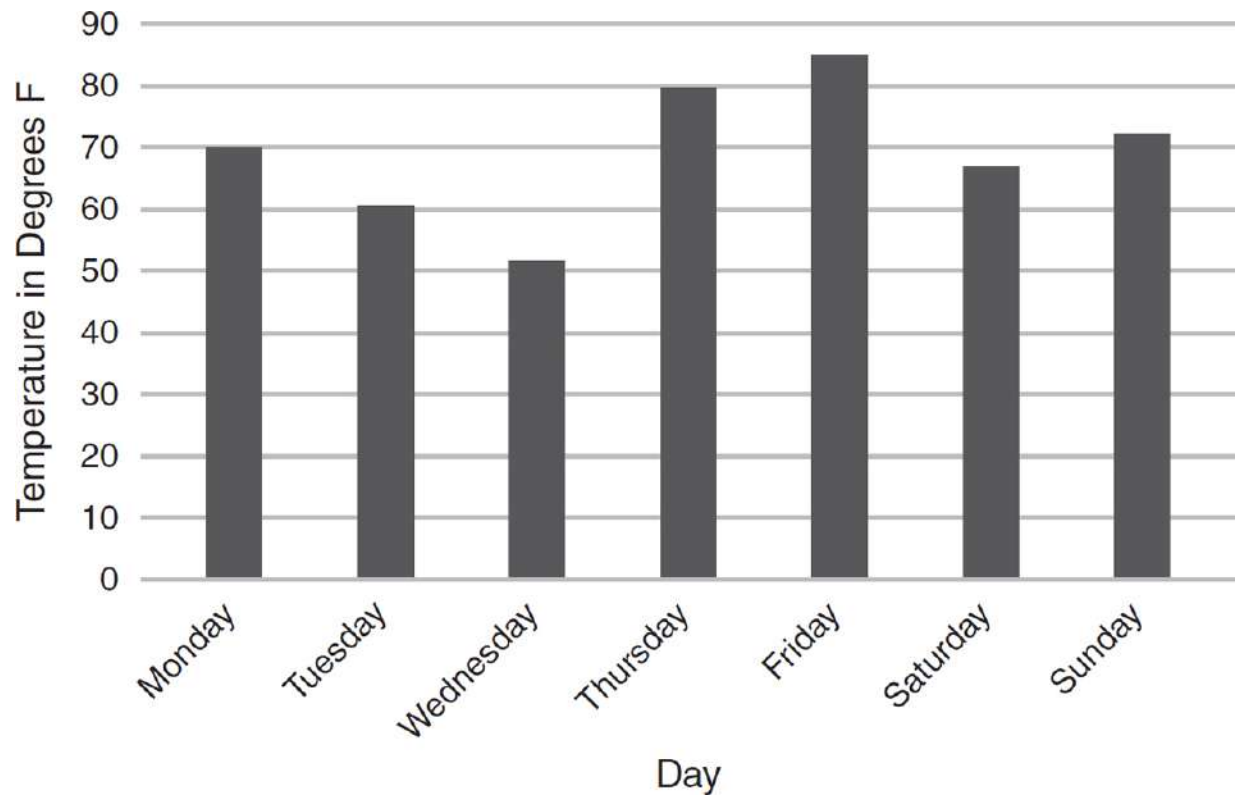


There are 6 points that fall underneath the line of best fit, so there are 6 commuters for whom the commute was shorter than predicted.

Histograms

Histograms are also known as **bar graphs**—they show the frequency of different values in a data set.

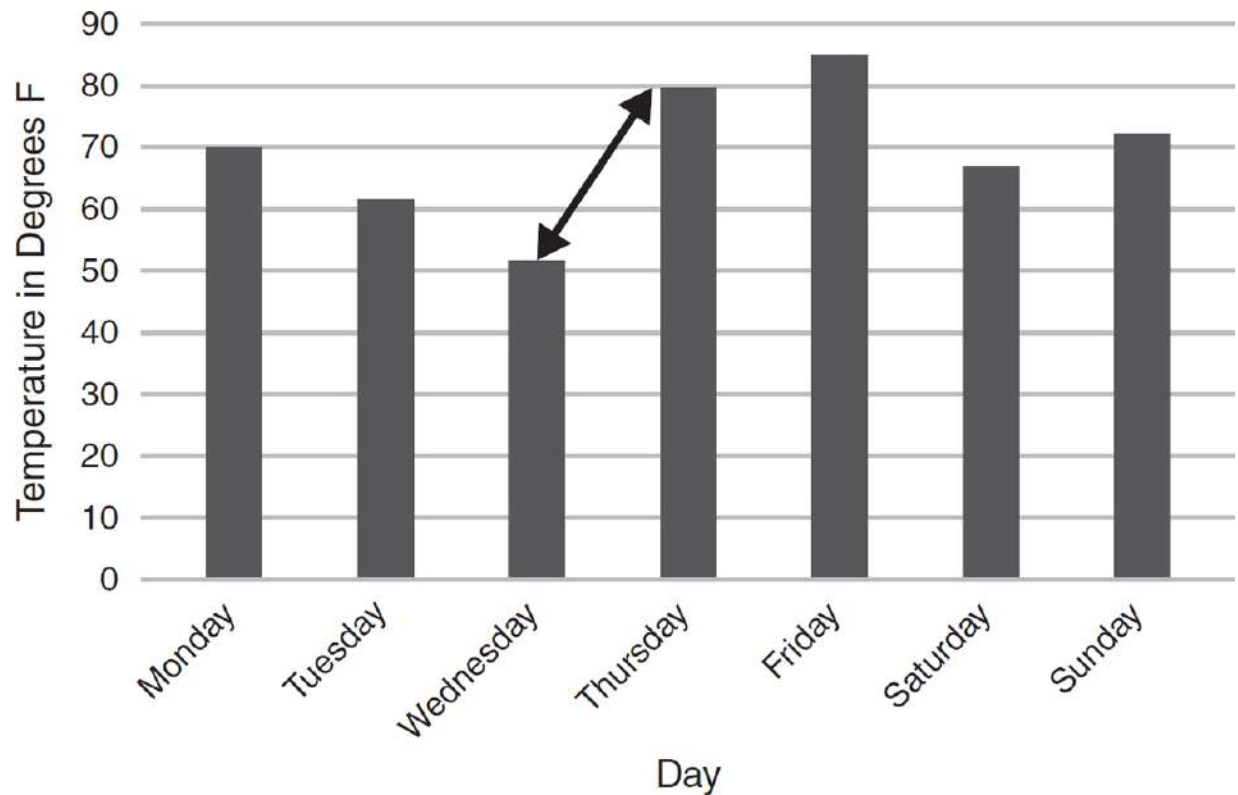
► Example



Based on the above graph, between what two days was there the largest increase in temperature?



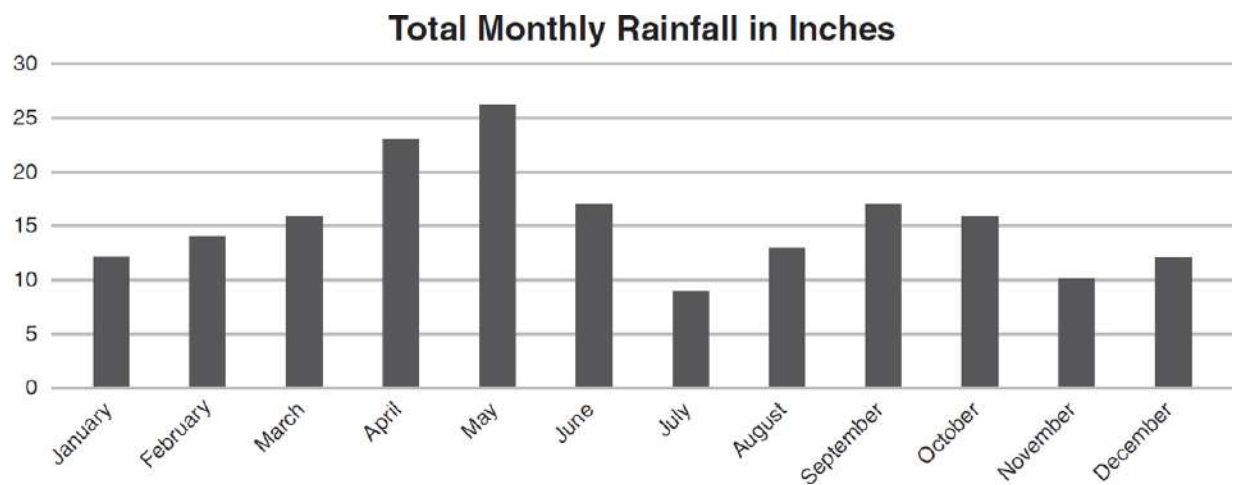
Solution



From Wednesday to Thursday, the temperature increases from approximately 52 degrees to 80 degrees. From the graph, this is clearly the largest increase from day to day.

Practice

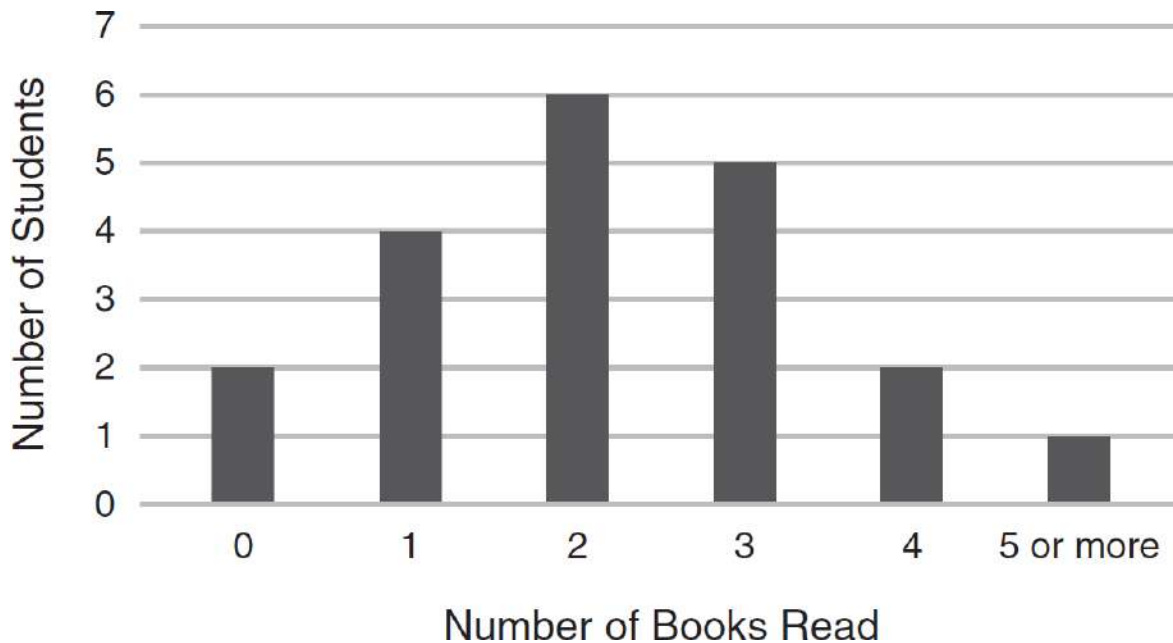
1.



During which months was the total monthly rainfall greater than 20 inches?

2. Which of the following best approximates the range in monthly rainfall over the course of a year?

- (A) 9
- (B) 17
- (C) 22
- (D) 26



The graph above shows the distribution of the number of 20 total students who read a certain number of books over the summer. Use this graph to answer questions 3–4.

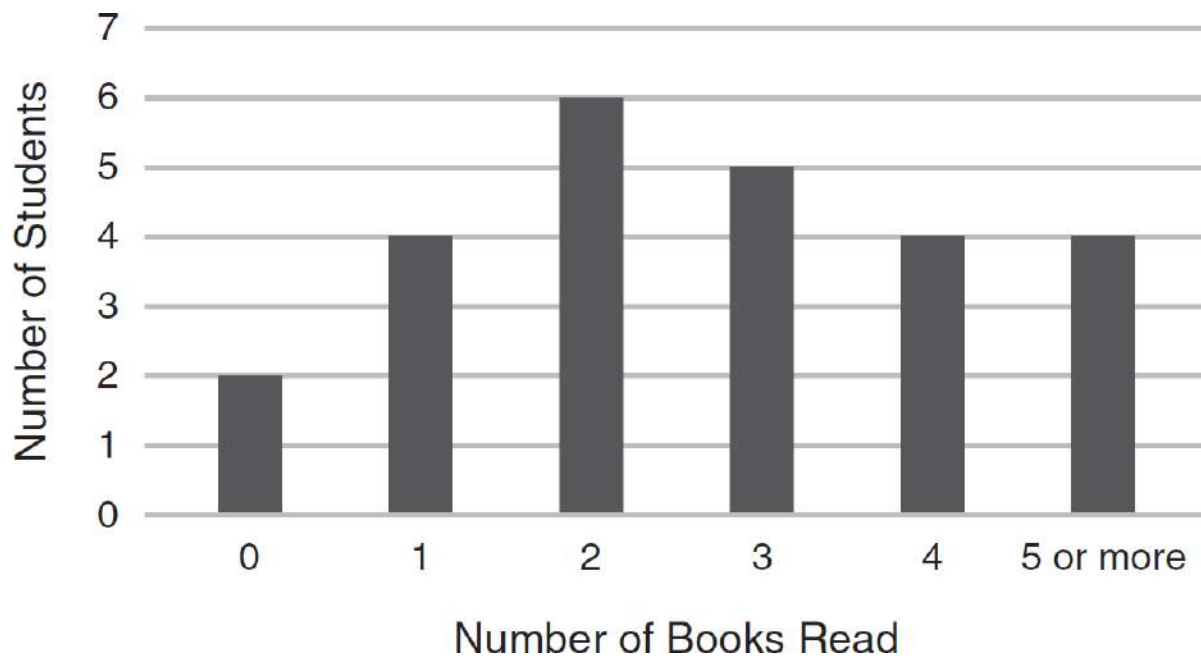
3. How many total students read just 2 or just 3 books?
4. If another student is added to the set who read 7 books over the summer, what will be the effect on the median number of books read and mean number of books read?

- (A) Median decreases, while the mean remains the same.
- (B) Both the median and the mean decrease.
- (C) Median remains the same, and the mean increases.
- (D) Both the median and the mean increase.

Solutions

April has rainfall of 23 inches, and May has rainfall of 26 inches, while all the other months have less than 20 inches of rain. So, just April and May would have total monthly rainfall greater than 20 inches.

The range of a data set is defined as the difference between the largest and smallest values in the set. May has the most rainfall at 26 inches, and July has the least rainfall at 9 inches. So, the difference between the two is $26 - 9 = 17$ giving a range for the data of choice **(B)**, 17.



The graph above shows the distribution of the number of students who read a certain number of books over the summer.

Six students read 2 books and 5 students read 3 books, so the total number of students who read just 2 or 3 books is $6 + 5 = 11$.

Given that there are 20 students total, the median will fall between the 10th and 11th terms in the set. In this case, the 10th and 11th terms are both students who read 2 books, so the median number of books read will not change even if a greater value is added to the set since a new median with 21 total students will remain at 2 books read. The mean, however, will change since it is a calculation of the average number of books read. An additional student who read 7 books would slightly increase the mean since the sum of all the books read would be greater. Thus, the answer is (C).

Tables

The SAT will frequently present data in a table form. When analyzing tables, be sure you clearly understand what the numbers in each cell represent. Do so by carefully matching the cells up with the corresponding descriptions.

► Example

Air Quality Index, or AQI, is a measure used to determine how safe it is to be outside given the air pollution that is in an area. The index ranges between 0 and 500, and measures of 151 or above are considered unsafe for the general public. Sixty cities in a country had their average AQI for two months measured, with the results tabulated below:

	AQI less than 151 in February	AQI greater than 151 in February	Total
AQI less than 151 in January	40	4	44
AQI greater than 151 in January	10	6	16
Total	50	10	60

What is the probability that one of the surveyed cities would have an unsafe AQI for the general population over both months surveyed?

✓ Solution

Let's be clear about what the numbers in the table represent (don't worry—you don't need to be this thorough for every problem; this is so you can clearly see how these tables work):

- 40 is the number of cities that had the lower AQI in both January and February.
- 10 is the number of cities that had a greater AQI in January and a lower AQI in February.
- 4 is the number of cities that had a greater AQI in February and a lower AQI in January.
- 6 is the number of cities that had a greater AQI in both January and February.
- 44 is the total number of cities that had a lower AQI in January.
- 16 is the total number of cities that had a higher AQI in January.
- 50 is the total number of cities that had a lower AQI in February.
- 10 is the total number of cities that had a higher AQI in February.
- There are 60 cities all together that were presented.

So, to calculate the probability that one of the surveyed cities would have an *unsafe* AQI (over 151) over *both* months, first see how many cities had the greater AQI in both January and February—there were 6 such cities. Then, divide this by the total number of cities presented, 60.

$$\frac{6}{60} = 0.1$$

So the probability that one of the surveyed cities would have an unsafe AQI over both months is

$$0.1.$$

Practice

Type of Math Class and Type of Class Instructor at a University

	Graduate Student	Professor	Totals
Lower-Level Class	80	40	120
Upper-Level Class	20	45	65
Totals	100	85	185

How many instructors in total taught lower-level classes?

How many instructors were there all together at the university?

What percent of all the math classes are taught by professors, calculated to the nearest whole percent?

If a math class is taught by a graduate student, what is the probability the class is an upper-level class?

Solutions

120. Match up the Lower-Level Class with the Totals and see that there are 120 total instructors for lower-level classes.

185. The bottom right cell shows how many total instructors of all types and all classes were at the university—185.

46%. There are a total of 185 classes, and out of these 185 classes, 85 are taught by professors. So, calculate the percent of classes taught by professors like this:

$$\frac{85}{185} \times 100 \approx 46\%$$

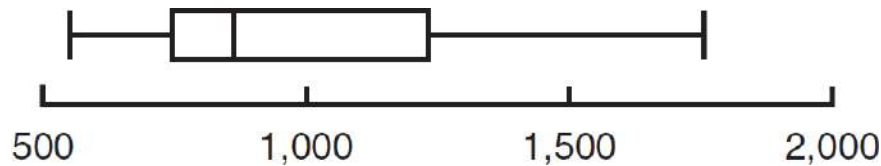
0.2. There are 100 total math classes taught by graduate students, and 20 of these classes are upper level. So, to calculate the probability, divide 20 by 100:

$$\frac{20}{100} = 0.2$$

Box Plots

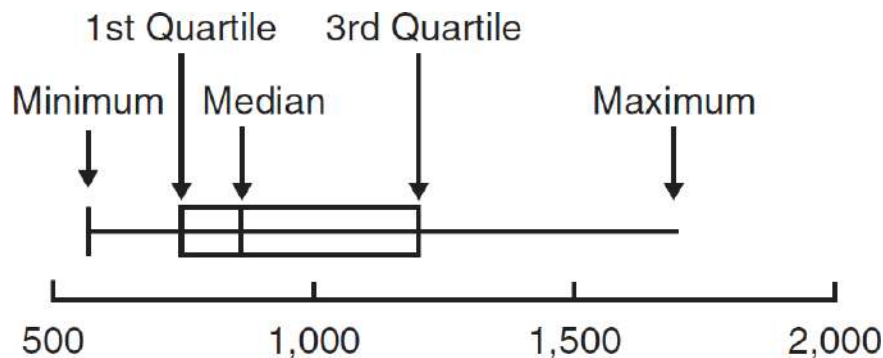
On occasion, you may come across box plots (a.k.a. whisker plots) on the SAT Math, called this because the two quartiles (fourths) of the data that are in the middle make a box (the least and most quartiles make “whiskers” on either side). These plots break up the values in the data set into four quadrants so you can more easily visualize the spread of the data and the median.

► Example



The box plot above summarizes the values for the average monthly apartment rental price in a particular neighborhood. Which of the following would be closest to the median monthly apartment rental price in this neighborhood?

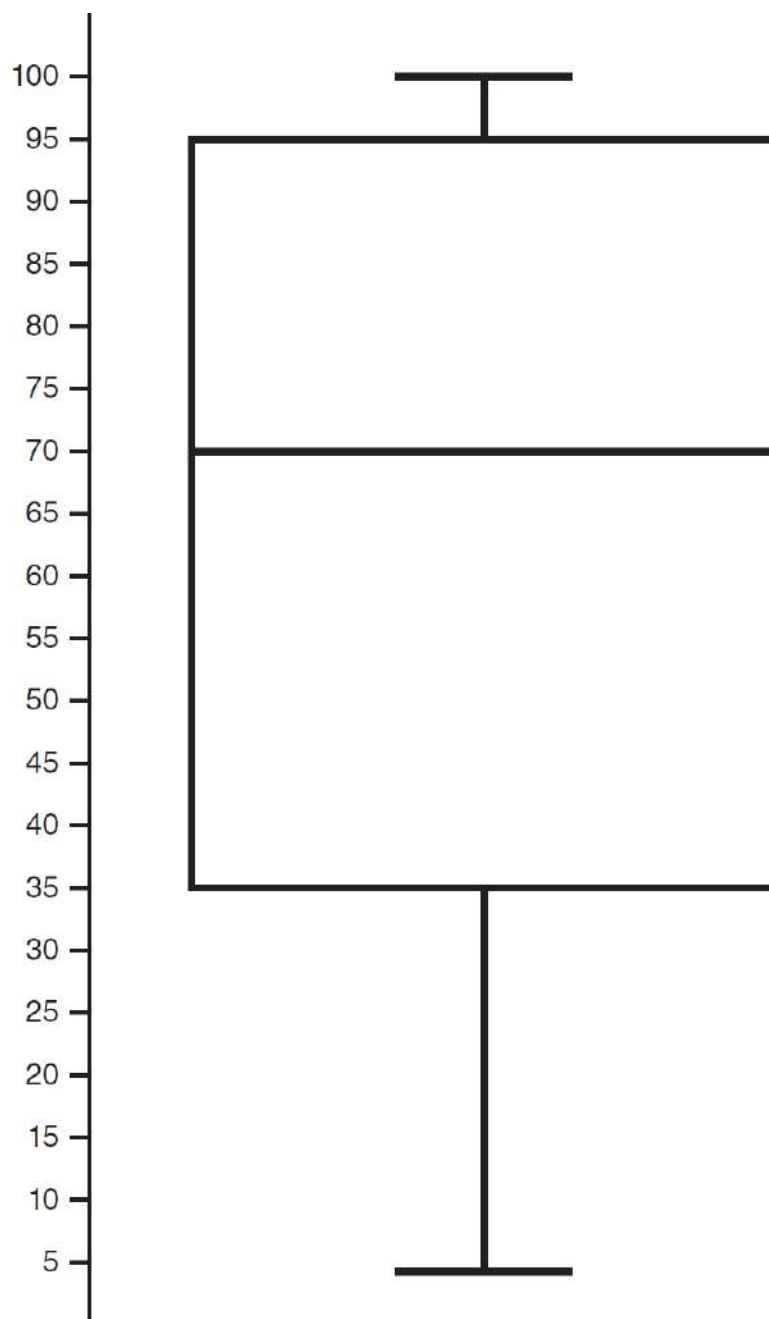
- (A) 500
- (B) 600
- (C) 800
- (D) 1,200



✓ Solution

From this, you can see that the median value would be approximately 800, making choice (C) the correct answer.

Practice



The above box plot summarizes the test results from a group of 300 seniors at a high school.

What is the minimum test score?

What is the median test score?

Which expression would give the test results, x , between the 50th and 75th percentiles?

- (A) $5 < x < 35$
- (B) $35 < x < 70$
- (C) $70 < x < 95$
- (D) $95 < x < 100$

Solutions

The very least value in the plot is 5.

In the middle of the box is the value 70, which would represent the median.

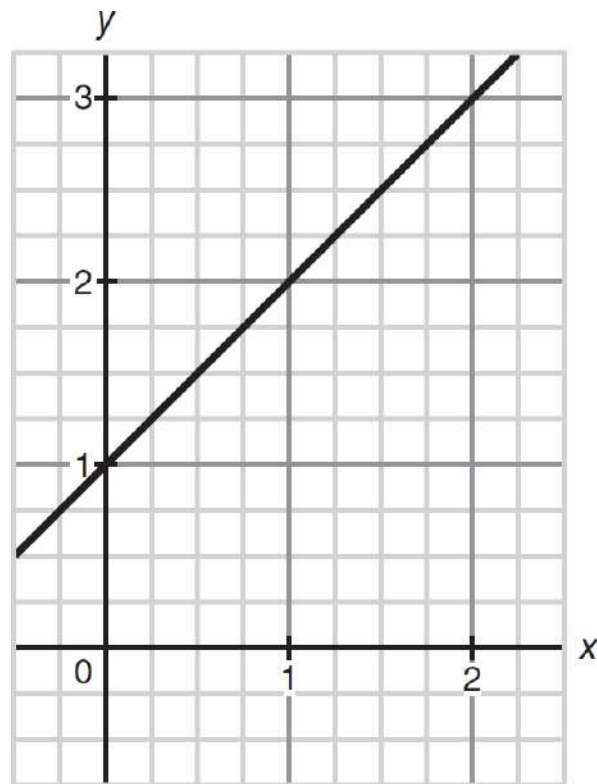
The test results between the 50th and 75th percentiles would be between the 2nd and 3rd quartiles. This would be between the median, 70, and the 3rd quartile, 95. So, the correct answer is choice (C).

Linear and Exponential Growth

A constant relationship between two variables is called a *linear relationship*. Linear *growth* occurs when the equation formed by the variables has a positive slope; as the x value increases, the y value also increases. Linear *decay* occurs when the equation formed by the variables has a negative slope; as the x value increases, the y value decreases.

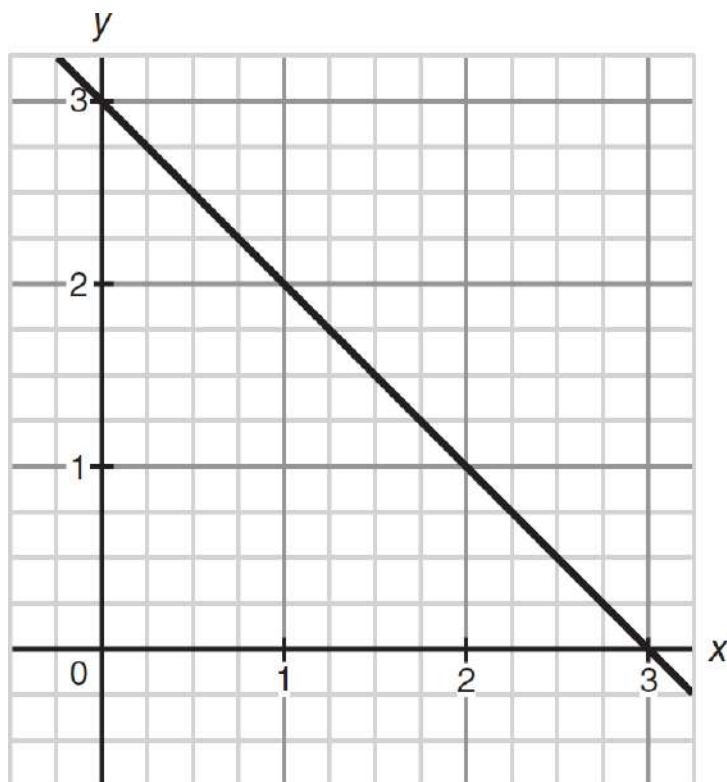
► Example 1

Linear growth for the equation $y = x + 1$



➤ Example 2

Linear decay for the equation $y = -x + 3$



A relationship between two variables x and y that is expressed in the form $y = a b^x$ or $y = a b^x + c$ is *exponential*. Exponential *growth* occurs when as the x value increases, the y value increases exponentially. Exponential *decay* occurs when as the x value increases, the y value decreases exponentially.

► Example 3

Twenty sheep were introduced to an island where they had no natural predators. The population of the sheep on the island over time is given in the table below.

Year	Number of Sheep
1800	20
1810	28
1820	38
1830	51
1840	65

If a function were made with the year as the x value and the number of sheep as $f(x)$, what would best describe the relationship between x and $f(x)$ if it were graphed in the xy -coordinate plane?

- (A) Exponential growth
- (B) Exponential decay
- (C) Linear growth
- (D) Linear decay

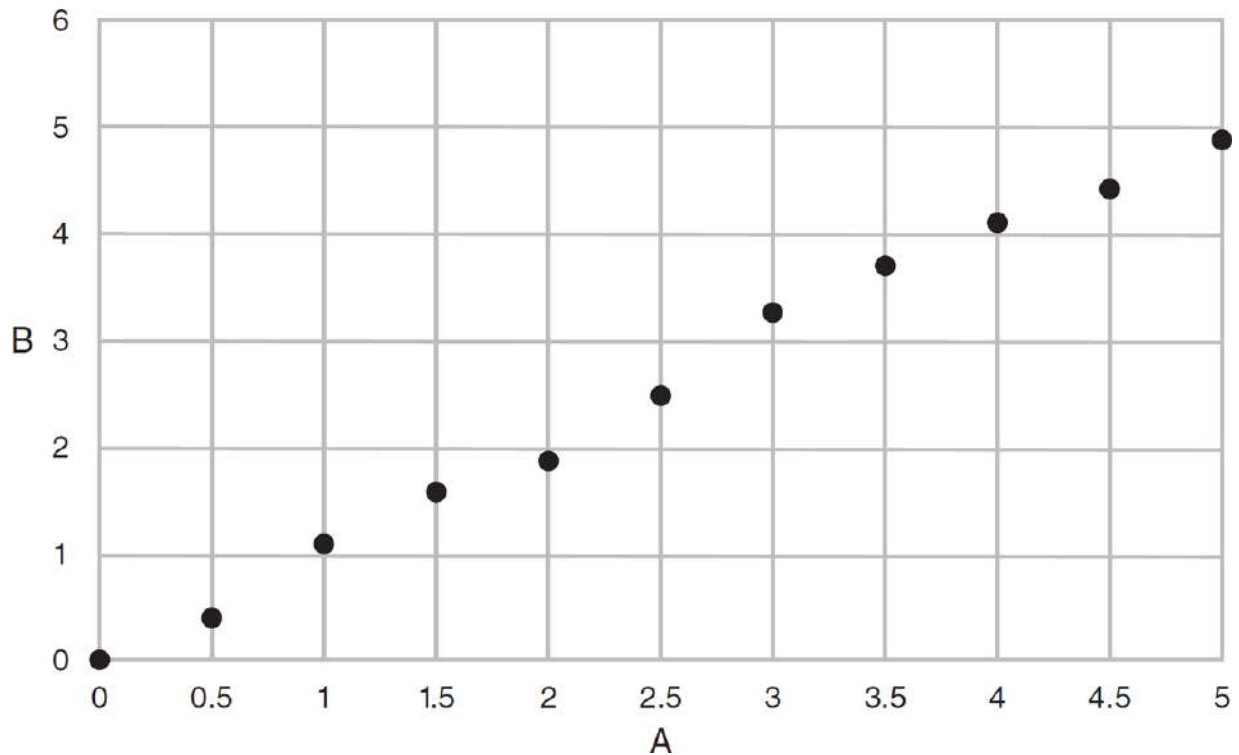
✓ Solution

Let's examine how much the number of sheep changes from year to year:

Year	Number of Sheep	Increase from 10 Years Prior
1800	20	Not applicable
1810	28	+8
1820	38	+10
1830	51	+13
1840	65	+14

We can see that not only is the number of sheep *increasing*, the number by which it is increasing in each interval is also *increasing*. If this had been linear growth, the number by which it increases in each interval would remain the same. So, the correct answer is choice **(A)**, exponential growth.

Practice



The line of best fit for the points given in the graph above would show what type of relationship between A and B ?

- A) Negative linear association
- B) Positive linear association
- C) Negative exponential association
- D) Positive exponential association

If the trend given in the above graph were to continue, what would be closest to the projected value of B when A is 6?

- A) 5
- B) 5.5
- C) 6
- D) 6.5

John wishes to place \$1,000 in an investment. Which of these circumstances would provide him with the greatest amount of money after a ten-year

period?

- A) An investment that grows linearly at 7% of the original amount each year
- B) An investment that has exactly \$60 added each year
- C) An investment in which 2% of the original amount and \$45 is added each year
- D) An investment that grows exponentially at 6% a year, compounded annually

Solutions

As the x values increase, the y values also increase at a steady rate. Therefore, this will be choice **(B)**, a positive linear association.

Be careful to notice that the increments on the x -axis are different from those on the y -axis. For each increase in 1 whole unit of x , the y value increases by about 1 whole unit. So, when A is 6, the B value would be 1 more than what it was at the point $(5, 5)$. So, the projected value for B would be choice **(C)**, 6, since the new point would be approximately $(6, 6)$.

Choices **(A)**, **(B)**, and **(C)** would all result in linear growth—a set amount of money would be added each year. Choice **(D)** is correct because with exponential growth, the amount by which the investment increases will *increase*, resulting in far greater growth over time.

Interpreting Constants and Functions

In order to assess your skill in understanding the meaning of functions, the SAT Math will frequently ask you to interpret constants and other components of functions.

► Example

Mary's investment in a stock portfolio, S , is modeled by the function

$$S(y) = 1000(1.15)^y$$

in which y is the number of years Mary holds the stock. What is the best interpretation of the number 1.15 in the context of this function?

- (A) When Mary holds the investment for 1.15 years, her overall investment increases by \$1,000.
- (B) When Mary holds the investment for 0.15 years, her overall investment decreases by \$1,000.
- (C) For each year that Mary owns the stock portfolio, her investment balance increases by \$15.
- (D) For each year that Mary owns the stock portfolio, her investment balance increases by 15% compounded annually.

✓ Solution

Use your knowledge of other concepts to help you interpret the meaning of constants in functions. Recall from the review of percentages that the formula for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, which has the same structure as the function $S(y) = 1000(1.15)^y$. The number 1.15 will correspond to $1 + \frac{r}{n}$. Since the intervals at which the investment value is compounded are one year, given the exponent y , $1 + \frac{r}{n}$ will equal $1 + \frac{0.15}{1} = 1.15$. This means that the percent increase each year is 15%. Therefore, the correct answer is choice **(D)**, since each year that she owns the portfolio, her investment balance will increase by 15%.

You could also use process of elimination to figure this out, since choices (A), (B), and (C) would all correspond to linear growth.

Practice

A chemist measured the amount of caffeine in different volumes of coffee:

Fluid Ounces	Milligrams of Caffeine
2	25
5	60
8	95

Which of the following functions best approximates the relationship between fluid ounces, F , and milligrams of caffeine, C ?

- A) $C = 4F$
- B) $C = 8F$
- C) $C = 12F$
- D) $C = 16F$

Given the relationship in the above table, which of the following would best approximate the milligrams of caffeine in 15 fluid ounces of coffee?

- A) 150
- B) 165
- C) 180
- D) 195

Questions 3–4 refer to the following information:

The half-life formula for a given substance is

$$N(t) = N_0 \left(\frac{1}{2} \right)^{\frac{t}{h}}$$

in which $N(t)$ is how much of the original amount of a substance, N_0 , remains after a certain amount of time t elapsed, and h is the half-life of the substance.

What would the value of $N(5)$ represent?

- A) How long it takes for half of a substance to decay
- B) How much of the substance would remain after 5 years had elapsed
- C) The number of years it takes for a substance to fully decay
- D) The original mass of a substance before calculation of its half-life

Suppose that a particular substance has a half-life of 20 years. What fraction of the substance in the year 2000 would remain in the year 2080?

- A) $\frac{1}{16}$
- B) $\frac{1}{8}$
- C) $\frac{1}{4}$
- D) $\frac{1}{2}$

Solutions

Glance ahead to the solutions to see that all of them are written in slope-intercept form. Determine which equation is correct by approximating the slope of the line. Let's use the points (2, 25) and (5, 60):

$$\frac{60 - 25}{5 - 2} = \frac{35}{3} \approx 11.67$$

This corresponds to choice (C), since 12 is the closest slope value to 11.67.

When there is an increase of 3 fluid ounces from 5 to 8, the milligrams of caffeine increases by 35, from 60 to 95. So, when going from 12 fluid ounces to 15 fluid ounces, the milligrams of caffeine will increase by a similar amount: $143 + 35 = 178$. This is closest to choice (C), 180.

Alternatively, you could have used the equation from the previous problem to solve for the number of milligrams, plugging in 15 for F .

$V(5)$ would represent the choice (B) since t is the amount of time that would go by, and if t is 5, then $N(5)$ would represent how much of the substance would remain after 5 half-lives had elapsed.

If the half-life is 20 years, according to the equation, half of the original amount of the substance would decay in a 20-year period. Since the time span is 80 years, going from 2000 to 2080, the original amount of the substance would be halved a total of 4 times:

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

So, the correct answer is (A) since there would be $\frac{1}{16}$ of the original amount.

Practice Drill #1—Problem Solving and Data Analysis Graphs and Data Interpretation

(If timing, take about 12 minutes to complete.)

The complete distribution of letter grades on a final exam are given in the table below:

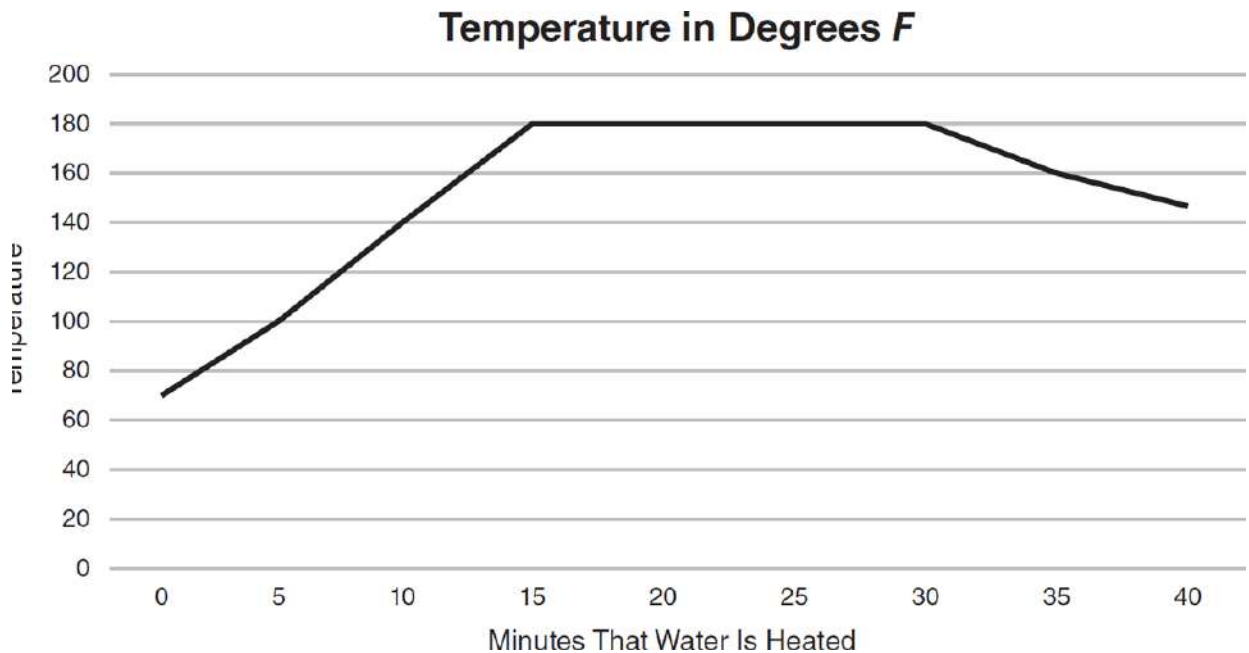
Letter Grade	Number of Students
A	12
B	16
C	10
D	7
F	5

What is the total number of students who completed the exam?

- A) 40
- B) 50
- C) 60
- D) 70

Which set of grades represents $\frac{3}{10}$ of the total number of students?

- A) A and B
- B) A and D
- C) C and D
- D) C and F



According to the graph above, what best approximates the initial temperature of the water?

- A) 70 degrees
- B) 90 degrees
- C) 100 degrees
- D) 120 degrees

Consider the graph above. A cook wanted to cook pasta during a 15-minute time period when the water temperature was steady. To accomplish this goal, between what times after starting to heat the water did the cook have the pasta in the water?

- A) 0–10 minutes
- B) 10–15 minutes
- C) 15–30 minutes
- D) 30–40 minutes

Questions 5–7 are about the following information.

A marketing agency surveyed clients from different online businesses to find the e-commerce conversion rate, i.e., what percentage of website visitors purchased something from the business. The results are compiled below:

Business Type	Conversion Rate
Industrial Supplies	4%
Pet Supplies	9%
Booksellers	10%
Electronics	26%
Toys	12%
Business Supplies	21%

Based on the above table, what is the probability that a visitor to an online bookstore will make a purchase?

- A) 0.1
- B) 0.2
- C) 0.3
- D) 0.4

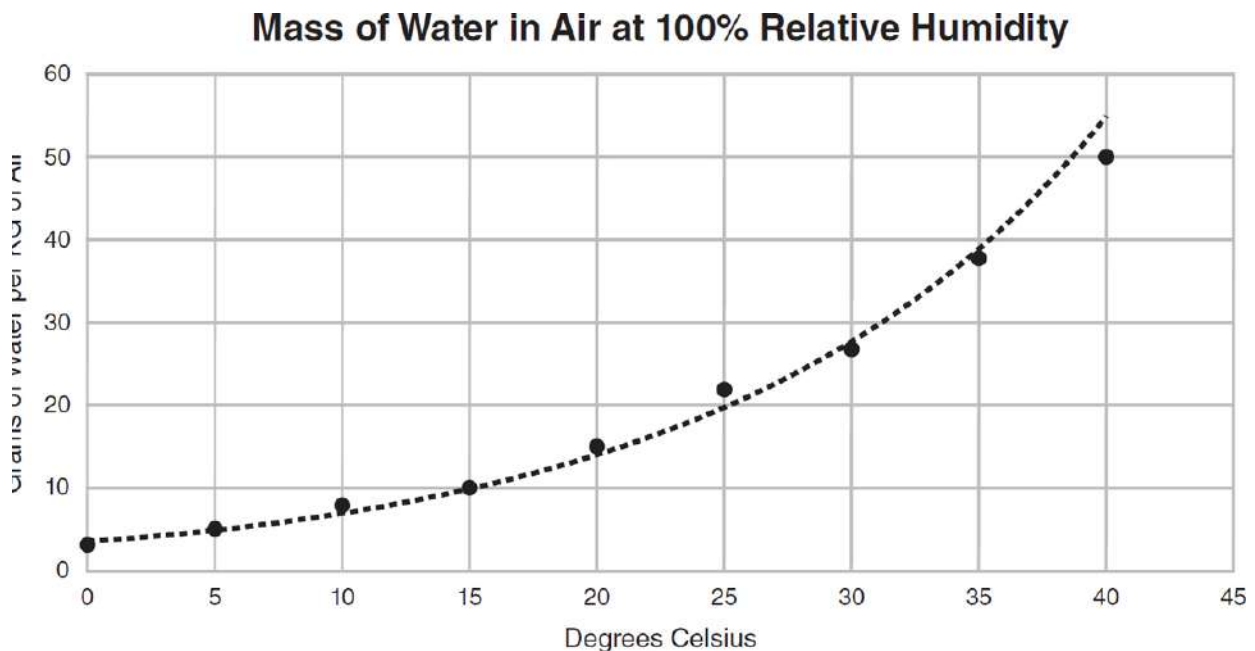
If an online electronics business has 2,000 visitors to its site in a month, how many of them would most likely purchase something?

- A) 440
- B) 520
- C) 650
- D) 860

What is the median conversion rate for the business types presented in the table?

- A) 5%

- B) 7%
- C) 9%
- D) 11%



The relationship between temperature and grams of water per kilogram of air in the above graph is best described as which of the following?

- A) Exponentially increasing
- B) Exponentially decreasing
- C) Linear increasing
- D) Linear decreasing

What is the predicted value of grams of water per kilogram of air at 25 degrees C?

- A) 16
- B) 20
- C) 24
- D) 28

The greatest outlier presented in the above graph is found at what temperature?

- A) 10 degrees
- B) 20 degrees
- C) 30 degrees
- D) 40 degrees

Solutions

B) Add up the total number of students from the right column to find the total number of students who completed the exam:

$$12 + 16 + 10 + 7 + 5 = 50$$

D) Find what $\frac{3}{10}$ of the total of 50 students would be:

$$\frac{3}{10} \times 50 = 15$$

Then, examine which combination will add up to 15. The only one that does is choice **(D)**, *C and F*, since $10 + 5 = 15$.

A) The initial temperature of the water is the temperature when 0 minutes have passed. At an x value of 0, the y value for temperature is closest to 70 degrees.

C) For the temperature to be steady, the line must be horizontal, having a slope of zero. Between 15 and 30 minutes, the line is horizontal, so it is in that range that the cook should cook the pasta.

A) Look at the percentage of conversion for booksellers—10%. Then, express this as a probability:

$$\frac{10}{100} = \frac{1}{10} = 0.1$$

B) The conversion rate for electronics businesses is 26% according to the table. So, find 26% of 2,000 to predict how many of the business's visitors will likely purchase something:

$$0.26 \times 2,000 = 520$$

D) Put the percentages in order from least to greatest:

$$4, 9, 10, 12, 21, 26$$

Since there are an even number of terms, and the two middle terms are different, find the median of the set by averaging 10 and 12:

$$\frac{10 + 12}{2} = 11$$

So, the median conversion rate is 11%.

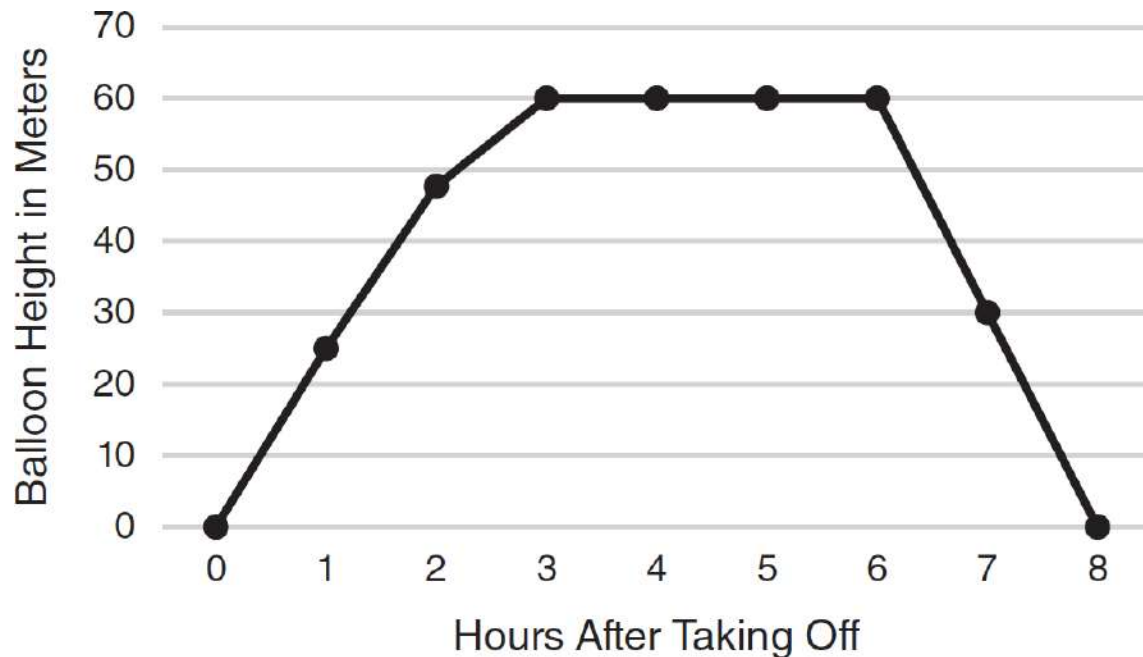
A) As the temperature increases, the grams of water per kg of air increase more and more. Had this been a constant increase, it would be linear. Since the increase accelerates, it would be an exponential increase.

B) The predicted value and measured value differ—the measured value is indicated by the dot, while the predicted value will be on the curved, dotted line. At 25 degrees Celsius, the grams of water per kilogram of air would be 20.

D) The greatest outlier is the value that most differs from the predicted value; in other words, it is the measured value that is farthest from the curve. At 40 degrees, the measured value of the grams is 50, while the curve would predict approximately 55. This point is farther away from the predicted values than any of the other dots presented in the graph.

Practice Drill #2—Problem Solving and Data Analysis Graphs and Data Interpretation

(If timing, take about 12 minutes to complete.)



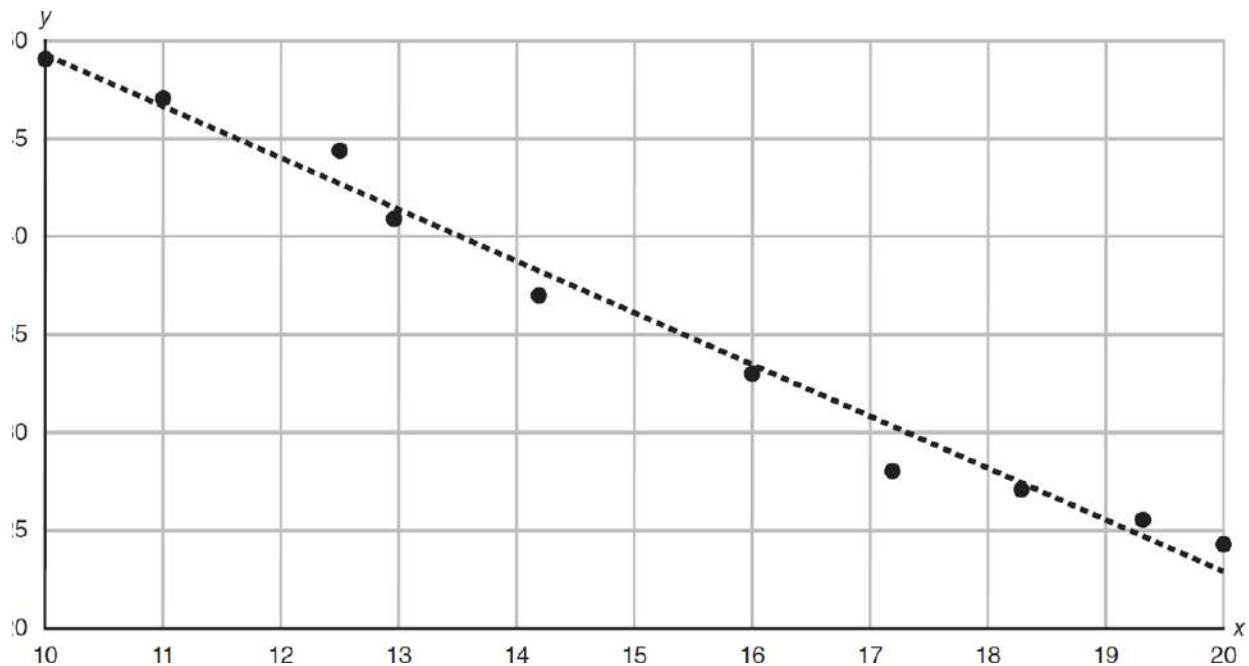
Sam took a balloon ride, in which he ascended to a height of 60 meters and remained there for a period of time. His height in meters above the ground as it relates to hours after taking off is graphed above.

For how many hours in his ride did he stay at his maximum height?

- A) 1
- B) 2
- C) 3
- D) 4

During what interval did Sam's height make the greatest change?

- A) Between 0–2 hours
- B) Between 2–4 hours
- C) Between 4–6 hours
- D) Between 6–8 hours



Consider the set of data in the scatterplot above.

Assuming the trend in the above graph were to continue, what would most likely be the y value of the predicted point with an x value of 25?

- A) 5
- B) 10
- C) 15
- D) 20

If a linear equation were made to represent this data, which of the following would most closely estimate its y -intercept?

- A) 48
- B) 60
- C) 75
- D) 90

A survey of 200 online shoppers asked respondents their favorite product to purchase on the internet. Their responses are summarized in the table below.

Item	Percent of Those Surveyed
Books/Music	17
Technology	18
Clothing	22
Online Education	6
Trips	11
Reused Goods	19
Other	7

Based on this table, how many of the 200 respondents primarily preferred reused goods, trips, or clothing?

- A) 104
- B) 132
- C) 150
- D) 178

Fund Name	Share Price	Expected Yearly Management Expenses per Share
Fund A	\$140	\$1.12
Fund B	\$80	\$0.64
Fund C	\$200	\$1.60
Fund D	\$64	\$0.51
Fund E	\$50	\$0.40

The table above shows the price per share and the estimated yearly expenses for five different actively managed mutual funds (investment products that purchase stocks in many different companies).

If Jennifer purchased 200 shares of Fund C, what would her yearly fund expenses most likely be?

- A) \$100
- B) \$190
- C) \$240
- D) \$320

Which function best approximates the relationship between the share price, S , and expenses, E , given that the two quantities have a linear relationship?

- A) $(0.008)S = E$
- B) $(0.08)S = E$
- C) $(0.8)S = E$
- D) $(8)S = E$

If William purchased 100 shares of Fund D, what inequality shows the percentage return on the investment that would allow him to make enough money to cover his expected yearly management expenses, calculated to the nearest tenth of a percent?

- A) Percent ≤ 0.8
- B) Percent ≥ 0.8
- C) Percent ≥ 1.2
- D) Percent ≤ 1.2



Figure 1

Assuming the trend of the best fit line in [Figure 1](#) continues, what would be the mass of flower A if it has a height of 55 cm?

- A) Approximately 12.5 grams
- B) Approximately 16 grams
- C) Approximately 19.5 grams
- D) Approximately 24 grams

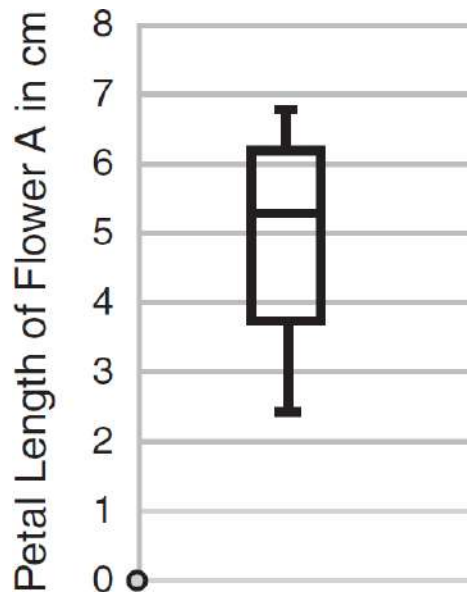


Figure 2

Based on Figure 2, in which of the following ranges would one find the median petal length of flower A?

- A) 3–4 cm
- B) 4–5 cm
- C) 5–6 cm
- D) 6–7 cm

Solutions

C) The maximum height of the balloon is at 60 meters, and Sam is at this height between 3 and 6 hours of flight, for a total of 3 hours.

D) During the interval between 6 and 8 hours, Sam's height decreased by 60 meters. This is a greater change than that of any of the other intervals, including between 0–2 hours when Sam's height increased by 50 meters.

B) Continue the same trend as illustrated in the graph. Since 25 is 5 more than 20, determine how much the y value decreases during a similar interval. Between x values of 15 and 20, the y value decreases from approximately 36

o 23, for a decrease of 13 units. So, to predict the y value when x is 20, subtract 13 from 23 (which is the y value when x is 20):

$$23 - 13 = 10$$

Therefore, the predicted value would be 10.

C) Be careful to examine the units of the graph—the x -axis begins at 10 rather than 0. So, to find the y -intercept, we need to determine what that point of intersection on the y -axis would be if we expanded the graph by 10 lesser x units. This could be accomplished by plugging in sample values, finding the slope of the equation, and then solving for the y -intercept. Fortunately, you can save time in this problem—determine how much the y value increases as the x value decreases from 20 to 10. This number will be identical to how much the y value increases as the x value decreases from 10 to 0, enabling us to find the y -intercept. Between 20 and 10, the y value increases from 23 to 49, for a total increase of $49 - 23 = 26$. So, increase the y value at the x value of 10, namely 49, by 26 units to find the y -intercept: $49 + 26 = 75$.

A) Add the percentages who preferred reused goods (19), trips (11), and clothing (22): $19\% + 11\% + 22\% = 52\%$. Then, take 52% of 200:

$$0.52 \times 200 = 104$$

D) Each share of Fund C has a per-share management expense of \$1.60. So, multiply 1.60 by 200 to find her likely yearly fund expenses:

$$1.6 \times 200 = 320$$

Therefore, her yearly fund expenses would most likely be \$320.

A) Find the relationship between share price and expenses by setting up an equation using sample values. Let's use a share price of 200 and a corresponding expense amount of 1.60, and call k the constant for which we are solving (since the terms have a linear relationship):

$$\begin{aligned} kS &= E \rightarrow \\ k(200) &= 1.60 \rightarrow \\ k &= \frac{1.60}{200} = 0.008 \end{aligned}$$

So, $(0.008)S = E$ would best approximate the relationship between the share price and expenses.

B) For a single share of Fund D, the percent of fund expenses are calculated as follows:

$$\frac{0.51}{64} \times 100 \approx 0.797$$

The percent of fund expenses would remain the same no matter how many shares William had. So, to express the percentage return on his investment that he would need to have to cover his expected yearly management expenses, William would need $\text{Percent} \geq 0.8$, since 0.8 would approximate 0.797 to the nearest tenth.

C) When the height of the flower increases from 45–50 centimeters, the mass of the flower increases by approximately 3 grams, going from 13.5 to 16.5 grams. So, increase the flower's mass when it is at 50 cm in height by a similar amount of 3 grams, since there is a steady, linear relationship between the flower's height and mass. $16.5 + 3 = 19.5$ grams, corresponding to choice **C**).

C) In a box plot, the line at which the two boxes meet is the median. In this graph, this occurs at approximately 5.25 cm. Therefore, the median will be within the range of 5–6 cm.

Geometry and Trigonometry

This section contains comprehensive review and strategies for all of the concepts below:

Area, Perimeter, and Volume

- Rectangle Area
- Triangle Area
- Perimeter of Polygons
- Parallelogram Area

- Trapezoid Area
- Surface Area
- Volume of a Box, Cylinder, Sphere, Cone, and Pyramid

Lines, Angles, and Triangles

- Supplementary Angles
- Vertical Angles
- Parallel Lines and a Transversal

Angles, Triangles, and Quadrilaterals

- Equilateral Triangles
- Isosceles Triangles
- Similar Triangles
- Angles in a Quadrilateral

Right Triangles and Trigonometry

- Pythagorean Theorem and Special Right Triangles
- Sine, Cosine, Tangent
- Area of an Equilateral Triangle
- Sine and Cosine of Complementary Angles

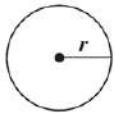
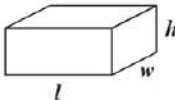
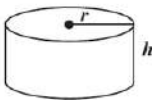
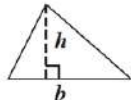
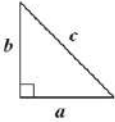
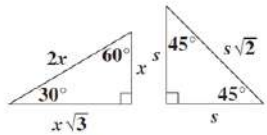
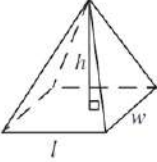
Circles

- Radius and Diameter
- Area and Circumference
- 360° in a Circle
- Inscribed Angle
- Arc Length
- Sector Area
- Circle Formula
- Radians

Area, Perimeter, and Volume

At the beginning of each math test section, the SAT will provide you with a formula sheet that covers much of what you will need for problems related

to area, perimeter, and volume.

 Radius of a circle = r Area of a circle = πr^2 Circumference of a circle = $2\pi r$	 Volume of a box = length $\times$ width $\times$ height = lwh
 Area of a rectangle = length $\times$ width = lw	 Volume of a cylinder = $\pi r^2 h$
 Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} bh$	 Volume of a sphere = $\frac{4}{3} \pi r^3$
 Pythagorean theorem: $a^2 + b^2 = c^2$	 Volume of a cone = $\frac{1}{3} \pi r^2 h$
 Special right triangles: 30-60-90 and 45-45-90	 Volume of a pyramid = $\frac{1}{3} \times \text{length} \times \text{width} \times \text{height} = \frac{1}{3} lwh$

KEY FACTS:

- A circle has 360 degrees.
- There are 2π radians in a circle.
- There are 180 degrees in a triangle.

SAT Math Strategy

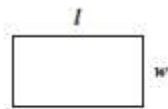
Even though many geometry formulas are provided, you should memorize these formulas so you have them at your fingertips. Memorizing the formulas will help you quickly remember

what formula you will need for a given problem. Also, if you memorize the formulas, you will have a better sense of what all the variables and constants mean and how to apply them.

As you can see from above, the SAT provides you with area formulas for two important shapes: the rectangle and the triangle.

Rectangle Area

The area of a rectangle is calculated as follows:



$$\text{Area of a rectangle} = \text{length} \times \text{width} = lw$$

► Example

What is the area of a rectangle with a length of 6 units and a width of 4 units?

✓ Solution

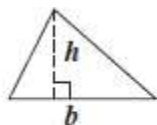
Multiply the length by the width to find the area:

$$6 \times 4 = 24$$

So, the area of the rectangle would be 24 square units.

Triangle Area

The SAT also provides you with the formula for the area of a triangle:

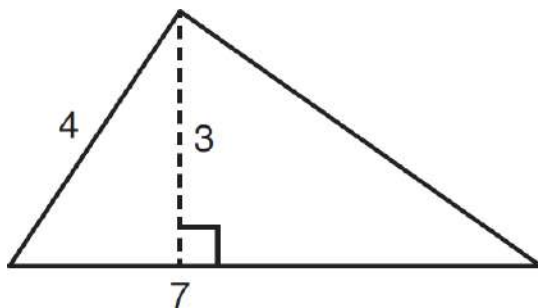


$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh$$

Note: In your math class, you may have learned this formula as $A = \frac{bh}{2}$. This formula is perfectly fine to use as well.

► Example

What is the area of the triangle drawn below?



✓ Solution

When calculating the area of triangles, parallelograms, and trapezoids, remember that the **height is perpendicular to the base**. On this particular problem, the base is 7 and the height is 3 (not 4). To calculate the area of the triangle, use the triangle area formula:

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{Base} \times \text{Height} \rightarrow \\ \text{Area} &= \frac{1}{2} \times 7 \times 3 = \frac{21}{2} = 10.5\end{aligned}$$

So, the triangle's area will be 10.5 square units.

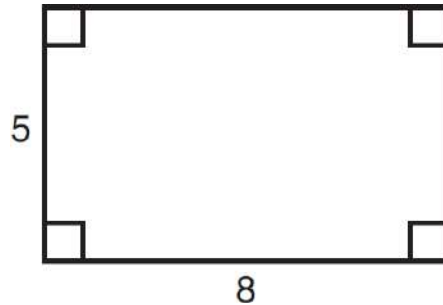
Now, let's consider the polygon concepts the SAT expects you to know, but for which you **do not** receive formulas.

Perimeter of Polygons

The *perimeter* of a shape is the length of its sides added together.

➤ Example 1

What is the perimeter of the rectangle below?

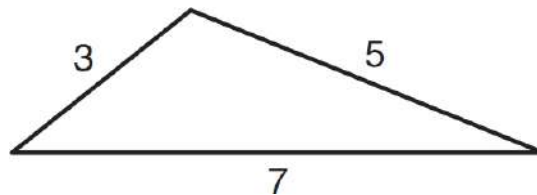


✓ Solution

In a rectangle, the sides opposite one another are equal. So the total length of the sides of the rectangle would be $5 + 5 + 8 + 8 = 26$ units.

➤ Example 2

What is the perimeter for the triangle below?



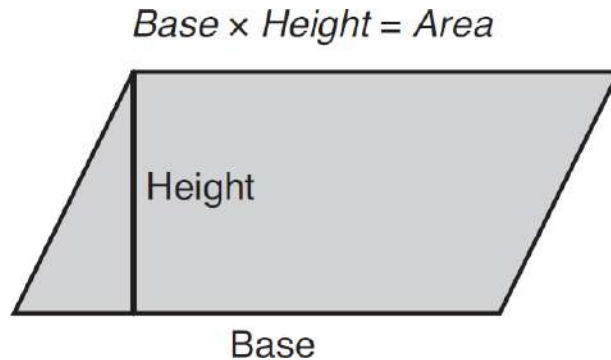
✓ Solution

Add the three side lengths of the triangle to find the perimeter:

$$3 + 5 + 7 = 15 \text{ units}$$

Parallelogram Area

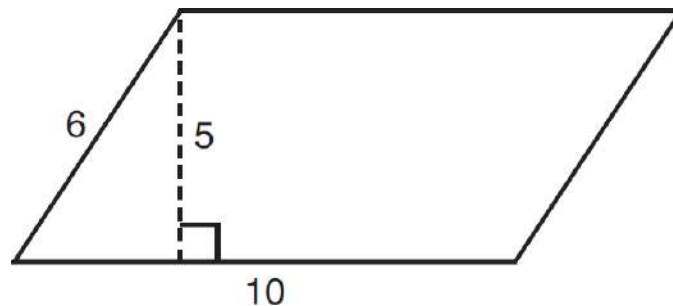
A parallelogram is a four-sided figure that has opposite sides that are parallel. (A rectangle is a special form of a parallelogram that has only 90° angles.) The area for a parallelogram is given below:



Be careful that you do not count one of the slanted edges as the height. Like with the triangle, the height will always be perpendicular to the base.

➤ Example

What is the area of the parallelogram drawn below?



✓ Solution

The base of the parallelogram is 10, and the height is 5 since it is perpendicular to the base. (The height is not 6 since this side is not perpendicular to the base.) So, the area is the base multiplied by the height:

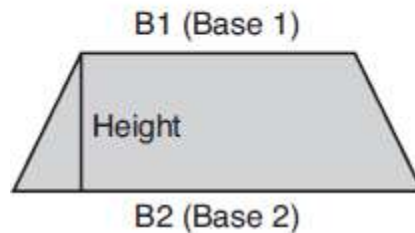
$$\text{Base} \times \text{Height} = 10 \times 5 = 50$$

The area is therefore 50 square units.

Trapezoid Area

A trapezoid is a four-sided figure that has a pair of opposite sides parallel to one another. The area for a trapezoid is calculated as follows:

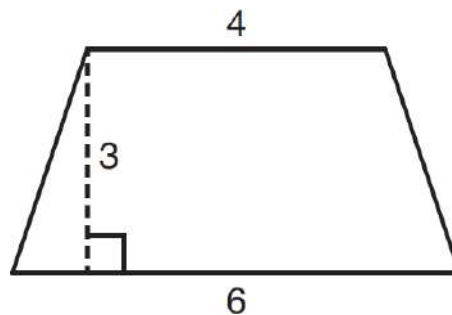
$$\frac{(B1 + B2)}{2} \text{ height} = \text{Area}$$



In essence, you are finding the “average” of the two bases and multiplying this average by the height of the trapezoid. As with the triangle and parallelogram, be mindful that the height must be perpendicular to the base.

► Example

What is the area of the trapezoid drawn below?



✓ Solution

Use 4 as B1, 6 as B2, and 3 as the height, plugging these values into the trapezoid formula:

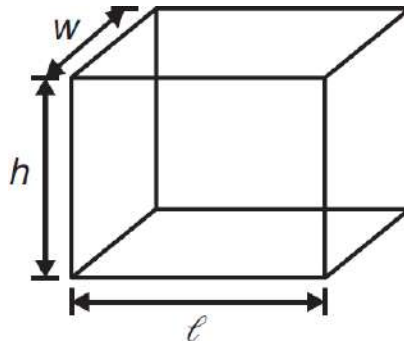
$$\frac{(B1 + B2)}{2} height = Area \rightarrow$$

$$\frac{(4 + 6)}{2} \times 3 = 5 \times 3 = 15$$

So, the area of the trapezoid is 15 square units.

Surface Area

The SAT may ask you to calculate the surface area of a *right rectangular prism*—this is a fancy way of saying *box*. To calculate the surface area of a box, take the area of each face of the box and add them together using this formula:



$$\text{Surface Area} = 2(lw) + 2(lh) + 2(wh)$$

► Example

What is the surface area of a right rectangular prism with a height of 3 units, a length of 5 units, and a width of 4 units?

✓ Solution

Carefully plug 3, 4, and 5 into the surface area formula to find the surface area:

$$\begin{aligned}
 \text{Surface Area} &= 2(lw) + 2(lh) + 2(wh) \rightarrow \\
 \text{Surface Area} &= 2(5 \times 4) + 2(5 \times 3) + 2(4 \times 3) \rightarrow \\
 &2(20) + 2(15) + 2(12) = \\
 &40 + 30 + 24 = 94
 \end{aligned}$$

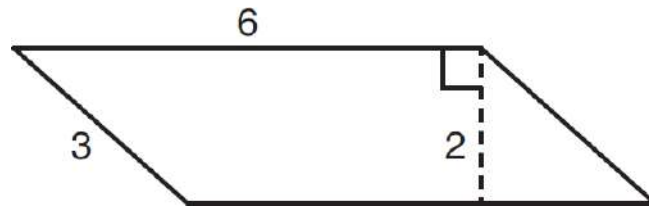
So, the surface area is 94 square units.

Practice

What is the area of a triangle with a height of 3 units and a base of 4 units?

What is the perimeter of a rectangle with a width of x and length of $2x$?

What is the area of the parallelogram below?



a trapezoid has a height h and an area of $6h$ square units, what would be the area of a trapezoid with bases of the same length but a height twice that of the original?

What is the surface area of a cube with an edge length of 3?

The perimeter of a rectangle is 24 inches, what would be the value of the length and the width of the rectangle in order to *maximize* the rectangle's area?

Solutions

$$\frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 4 \times 3 = 6$$

There are two sides of length x and two sides of length $2x$:

$$2(x) + 2(2x) = 2x + 4x = 6x$$

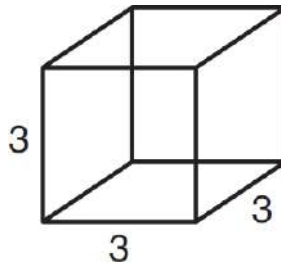
$$\text{Base} \times \text{Height} = 6 \times 2 = 12$$

Consider the area formula for a trapezoid:

$$\frac{(B_1 + B_2)}{2} \text{height} = \text{Area}$$

the bases remain the same, and the height is doubled, the original area will be doubled since the only thing that changes is the height. So, the new area could be $12h$.

For a cube, all the edges are of the same length:



So, plug 3 in for length, width, and height in the surface area formula:

$$\text{Surface Area} = 2(lw) + 2(lh) + 2(wh) \rightarrow$$

$$\text{Surface Area} = 2(3^2) + 2(3^2) + 2(3^2) \rightarrow$$

$$\text{Surface Area} = 18 + 18 + 18 = 54$$

Alternatively, you could calculate the surface area for one of the sides, $3 \times 3 = 9$, and multiply this by 6 since there are 6 total sides: $6 \times 9 = 54$ square units.

To maximize the area of the rectangle while minimizing the perimeter, the rectangle should be a *square*. You can see why this would be by trying out some sample values for length and width (that would allow for a perimeter of 24):

Length: 6 inches, width: 6 inches. Area is 36 square inches.

Length: 7 inches, width 5 inches. Area is 35 square inches.

Length: 8 inches, width 4 inches. Area is 32 square inches.

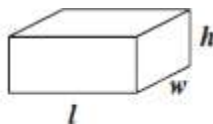
Length: 9 inches, width 3 inches. Area is 27 square inches.

So, the length and width should each be 6 inches in order to maximize the area of the rectangle.

Volume Calculations

The volume formulas you need to know are all provided at the beginning of each math section. Here are examples showing how to apply each of these formulas.

Volume of a Box (Right Rectangular Prism)



$$\text{Volume of a box} = \text{length} \times \text{width} \times \text{height} = lwh$$

A common error students make is to confuse area and volume. Area will only be a calculation of two dimensions, like $\text{length} \times \text{width}$. Volume will be a calculation of three dimensions, as we can see above.

➤ Example

What is the volume of box with a length $2x$ inches, a width of $3x$ inches, and a height of y inches?

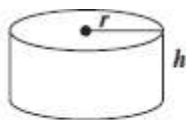
✓ Solution

Even though this asks you to calculate expressions with x and y , you can still use the same formula as above and multiply the length, width, and height together.

$$lwh = (2x)(3x)(y) = 6x^2y$$

So, the volume would be $6x^2y$ cubic inches.

Volume of a Cylinder

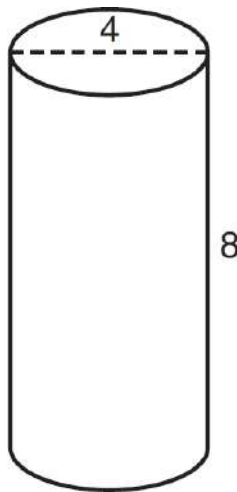


$$\text{Volume of a cylinder} = \pi r^2 h$$

Think about calculating the volume of a cylinder as taking the area of the circle on top, then multiplying it by the height all the way down.

➤ Example

What is the volume of a cylinder with a height of 8 inches and a diameter of 4 inches, as shown below?



✓ Solution

Be careful to distinguish between the radius and the diameter. The radius is *half* the length of the diameter, so the radius for the above cylinder would be 2. Then, plug 2 in for the radius and 8 in for the height to solve for the volume:

$$\pi r^2 h = \pi(2)^2(8) = 32\pi$$

So, the volume would be 32π cubic inches.

Volume of a Sphere



$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

➤ Example

Consider the volume of a sphere with a radius of x units. How much greater will the volume of a sphere with double the radius be?

✓ Solution

The volume of the original sphere will be $\frac{4}{3}\pi x^3$ cubic units. The radius of the second sphere will be $2x$, so its volume will be $\frac{4}{3}\pi(2x)^3 = \frac{4}{3}\pi(8x^3)$ cubic units. Divide the second volume by the first to see the multiple by which it is greater than the first:

$$\frac{\frac{4}{3}\pi(8x^3)}{\frac{4}{3}\pi x^3} \rightarrow \frac{\frac{4}{3}\pi(8x^3)}{\frac{4}{3}\pi x^3} = 8$$

So, the volume of the second sphere will be 8 times greater than the first. If you realize that the only thing that will change from one sphere to the next is the radius, you can simply calculate the ratio of the cubes of the radii to arrive at your solution.

Volume of a Cone



$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

➤ Example

How many cones of with radius 4 and height 6 will have the same total volume as a cylinder with the same radius and height?

✓ Solution

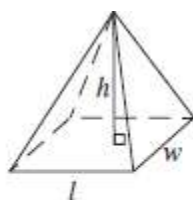
The SAT will frequently test your proficiency in manipulating algebraic expressions instead of doing long calculations. This question is a great example of this. Rather than calculating the volume of each cone, look at the relationship between the two formulas:

$$\text{Cone Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Cylinder Volume} = \pi r^2 h$$

The cylinder volume is three times that of the cone volume, since everything in the two formulas is the same except for the $\frac{1}{3}$ in front of the cone volume formula. Since $\frac{1}{3}$ goes into 1 three times, three cones could fit into a cylinder with the same radius and height as the cone.

Volume of a Pyramid



Volume of a pyramid =

$$\frac{1}{3} \times \text{length} \times \text{width} \times \text{height} = \frac{1}{3}lwh$$

► Example

Consider a pyramid with a volume of 20 cubic inches. If the width is cut in half and the height is doubled, what would be the volume of the resulting pyramid?

✓ Solution

Focus on how the changes to the width and height would affect the volume calculation.

For the original pyramid, the volume calculation would be as follows:

$$20 = \frac{1}{3}lwh$$

For the second pyramid, the new width is $\frac{1}{2}w$ and the new height is $2h$. So, the volume would be:

$$\frac{1}{3}l\left(\frac{1}{2}w\right)(2h) = \frac{1}{3}l\left(\frac{1}{2}w\right)(2h) = \frac{1}{3}lwh$$

Therefore, the volume of the new pyramid will be identical to the volume of the first: 20 cubic inches.

Practice

What is the volume of a box with dimensions 3 inches by 2 inches by 5 inches?

What is the volume of a cone with a radius and height both of y centimeters?

What is the volume of a sphere with a diameter of 6 inches?

How many pyramids with dimensions 1 by 1 by 1 inch would fit in a box with dimensions 1 inch by 2 inches by 2 inches?

If the height of a cylinder were tripled and its radius halved, what would the volume of the new cylinder be as a multiple of the volume of the original?

Solutions

$V_h = 3 \times 2 \times 5 = 30$ cubic inches

cone Volume $= \frac{1}{3}\pi r^2 h \rightarrow \frac{1}{3}\pi y^2 y = \frac{1}{3}\pi y^3$

Be sure to use the radius and not the diameter—the radius for this sphere is 3 inches. Plug 3 in to the sphere volume formula:

$$\text{Volume} = \frac{4}{3}\pi r^3 \rightarrow \frac{4}{3}\pi 3^3 = \left(\frac{4}{3}\right)(27)\pi = 36\pi \text{ cubic inches}$$

Divide the volume of a 1 by 2 by 2 inch pyramid by that of a 1 by 1 by 1 inch pyramid to see how many of the smaller pyramids would fit within the larger one.

$$\frac{\left(\frac{1}{3}(1 \times 2 \times 2)\right)}{\left(\frac{1}{3}(1 \times 1 \times 1)\right)} = \frac{\frac{1}{3} \times 4}{\frac{1}{3} \times 1} = \frac{\frac{4}{3}}{\frac{1}{3}} = \frac{4}{1} = 4$$

o, four of the smaller pyramids would fit in the larger pyramid.

se the formula for Cylinder Volume = $\pi r^2 h$. For the original cylinder, let's st use r as the radius and h as the height, making the volume $\pi r^2 h$. For the ew cylinder, the height is tripled and the radius is halved. So, use $3h$ for the ight and $\frac{1}{2}r$ for the radius and calculate the new volume:

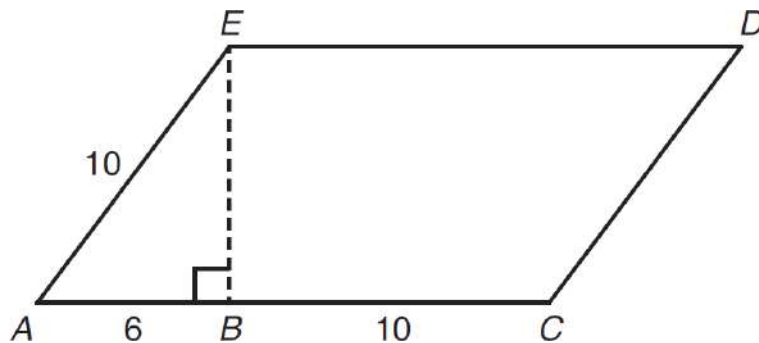
$$\pi \left(\frac{1}{2}r\right)^2 (3h) = \pi \left(\frac{1}{4}r^2\right)(3h) = \frac{3}{4}\pi r^2 h$$

o, the volume of the new cylinder would be $\frac{3}{4}$ the volume of the original one.

Practice Drill—Area, Perimeter, and Volume

(If timing, take about 12 minutes to complete.)

Fill-In Practice: Write your answer in the underlined blank under each question.



$AEDC$ is a parallelogram, with the length of $\overline{AB}$ 6 units, the length of $\overline{BC}$ 10 units, and the length of $\overline{AE}$ 10 units. Point B is on $\overline{AC}$ and $\overline{BE}$ is perpendicular to $\overline{AC}$. What is the area of the parallelogram in square units?

Answer: _____

A rectangular plot of land has a length twice its width. If the perimeter of the rectangle is 90 feet, what is its area in square feet?

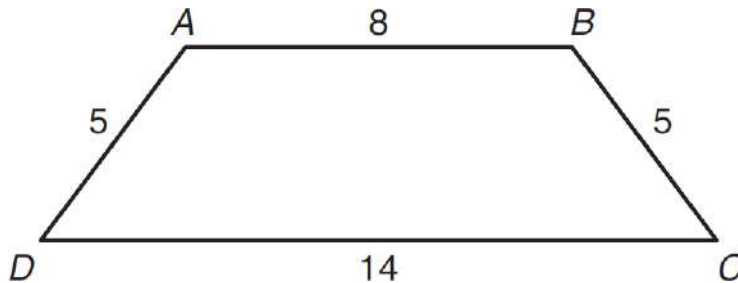
Answer: _____

If a cube has a volume of 125 cubic centimeters, what is the combined length of all the edges of the cube in centimeters?

Answer: _____

A cylinder has a volume of 150π square units and a height of 6 units. What is the diameter of the base of the cylinder in units?

Answer: _____



What is the area in square units of the trapezoid above, in which $\overline{AB}$ and $\overline{CD}$ are parallel?

Answer: _____

Multiple-Choice Practice

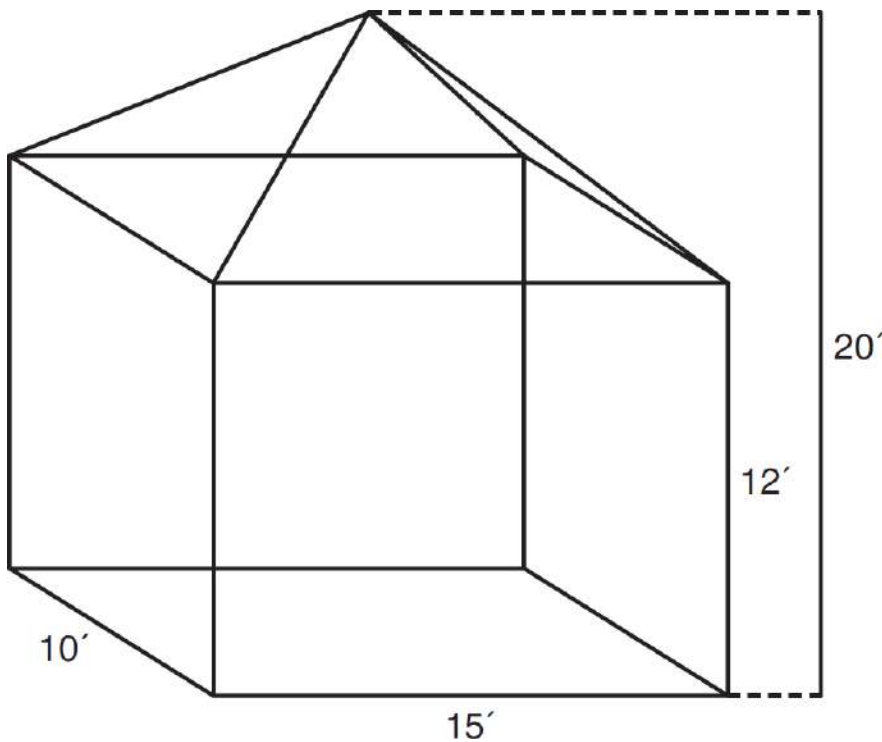
Each gallon of wall paint that Julius purchases covers 400 square feet. If Julius is going to paint 3 walls, each with dimensions 40 feet by 12 feet, what is the

minimum number of gallons of paint he needs to purchase, assuming he can only buy whole gallons and that there are no windows or other irregularities on the walls?

- a) 2
- b) 3
- c) 4
- d) 5

A basket maker designs cylindrical baskets with a radius of x centimeters, for which the height of the basket is twice the radius of the basket. Which of the following would correctly express the volume, V , in cubic centimeters, of such baskets?

- a) $V = 2\pi x^3$
- b) $V = \pi x^2$
- c) $V = 4\pi x^2$
- d) $V = 4\pi x^3$



the three-dimensional figure above, a rectangular prism with dimensions 12 feet by 10 feet by 15 feet has a pyramid with a base of 10 feet by 15 feet on top of it. The height of the rectangular prism and pyramid together is 20 feet. What is the volume of this entire figure?

- A) 900 cubic feet
- B) 1,460 cubic feet
- C) 2,200 cubic feet
- D) 3,000 cubic feet

A shoebox in the shape of a rectangular prism is supposed to have a volume between 432 and 504 cubic inches, inclusive. The box needs to have a length of 12 inches and a width of 6 inches. What is the range for a possible value for the height of the box?

- A) 5–6 inches inclusive
- B) 6–7 inches inclusive
- C) 7–8 inches inclusive
- D) 8–9 inches inclusive

A cylindrical tank is 3 meters deep and has a diameter of 10 meters. If the density of water is $997 \frac{\text{kg}}{\text{m}^3}$, how would one correctly calculate the mass in kilograms of the water in the tank if it is filled to the top with nothing but water?

- A) $\pi \times (10^2) \times 3 \times 997$
- B) $\pi \times (10^2) \times (1.5) \times 997$
- C) $\pi \times (5^2) \times (1.5) \times 997$
- D) $\pi \times (5^2) \times 3 \times 997$

Solutions

1. **(128)** $\overline{BE}$ is the height of the parallelogram, so use the Pythagorean theorem to solve for its length:

$$\begin{aligned}a^2 + b^2 &= c^2 \rightarrow \\6^2 + b^2 &= 10^2 \rightarrow \\36 + b^2 &= 100 \rightarrow \\b^2 &= 64 \rightarrow b = 8\end{aligned}$$

Then, multiply the height 8 by the base of 16 (add $\overline{AB}$ and $\overline{BC}$) to find the area of the parallelogram:

$$8 \times 16 = 128$$

2. **(450)** Let's call x the length of the rectangle and y the width of the rectangle. Since the length is twice that of the width, we can say that $x = 2y$. The perimeter of the rectangle is $2x + 2y$, and since $x = 2y$, we can use substitution to solve for either the width or length—let's solve for the width:

$$\begin{aligned}2x + 2y &= 90 \rightarrow \\2(2y) + 2y &= 90 \rightarrow \\4y + 2y &= 90 \rightarrow \\6y &= 90 \rightarrow \\y &= 15\end{aligned}$$

So, the width of the rectangle is 15, and the length would be twice 15: 30. Then multiply 30 and 15 together to find the area of the rectangle:

$$15 \times 30 = 450$$

3. **(60)** The volume, V , of a cube of side length x is $V = x^3$. Solve for x given that the volume is 125 cubic centimeters:

$$\begin{aligned}
 V &= x^3 \rightarrow \\
 125 &= x^3 \rightarrow \\
 \sqrt[3]{125} &= x \rightarrow \\
 5 &= x
 \end{aligned}$$

There are 12 total edges in a cube, so multiply 5 by 12 to find the combined length of all the edges of the cube:

$$5 \times 12 = 60$$

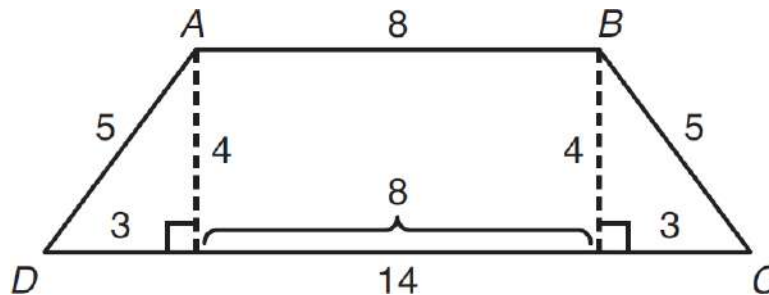
4. **(10)** Plug the volume and the height into the cylinder volume formula:

$$\begin{aligned}
 \text{Cylinder Volume} &= \pi r^2 h \rightarrow \\
 150\pi &= \pi r^2 (6) \rightarrow \\
 150 &= r^2 (6) \rightarrow \\
 25 &= r^2 \rightarrow r = 5
 \end{aligned}$$

Now, double the radius to solve for the base diameter:

$$5 \times 2 = 10$$

5. **(44)** First, determine the height of the trapezoid. The trapezoid is isosceles which makes it easier to find the height using the Pythagorean theorem, since $3^2 + 4^2 = 5^2$.



Now, use the area formula for a trapezoid to solve:

$$\frac{(B1 + B2)}{2} height = Area \rightarrow$$

$$\frac{(8 + 14)}{2}(4) = \frac{22}{2}(4) = 11 \times 4 = 44$$

6. **(C)** Find the surface area of each individual wall:

$$40 \times 12 = 480$$

Then multiply 480 by 3 to get the total area of the three walls:

$$480 \times 3 = 1,440$$

Now, divide 1,440 by 400 to see how many gallons of paint would be needed:

$$\frac{1,440}{400} = 3.6$$

Since Julius cannot purchase a partial gallon of paint, he must buy 4 full gallons to accomplish his task.

7. **(A)** Since the height of the basket is twice the radius of the basket, the height will be $2x$. Plug x in for the radius and $2x$ in for the height to express the volume:

$$V = \pi r^2 h \rightarrow$$

$$V = \pi x^2 (2x) \rightarrow$$

$$V = 2\pi x^3$$

8. **(C)** The figure is a box with dimensions 12 by 10 by 15 with a pyramid on top of it. The pyramid has a base of 10 by 15 and a height of 8, since $20 - 12 = 8$. Find the volumes of these two figures and add them together. First, the volume of the box:

$$lwh = 10 \times 15 \times 12 = 1,800$$

Next, the volume of the pyramid:

$$V = \frac{1}{3}lwh = \frac{1}{3} \times 10 \times 15 \times 8 = 400$$

Finally, add these separate volumes together to find the total volume:

$$1,800 + 400 = 2,200$$

9. **(B)** Pick a value that is within the range of 432 and 504 and solve for the possible height. Halfway between 432 and 504 is 468, so we can use that as a sample volume. The length and width are given, so let's plug in 468 for the volume and solve for the potential height:

$$\begin{aligned} V &= lwh \rightarrow \\ 468 &= 12 \times 6 \times h \rightarrow \\ \frac{468}{(12 \times 6)} &= 6.5 = h \end{aligned}$$

The only option that has 6.5 as a possibility for the height is choice **(B)**.

10. **(D)** Multiply the density by the volume of the tank so we can find the mass of water in the tank—when doing this, we are left with kilograms as the resulting unit. Use 5 meters as the radius, since radius is half the diameter:

$$\begin{aligned} \pi r^2 h \times \text{Density} &\rightarrow \\ \pi \times (5^2) \times 3 \times 997 & \end{aligned}$$

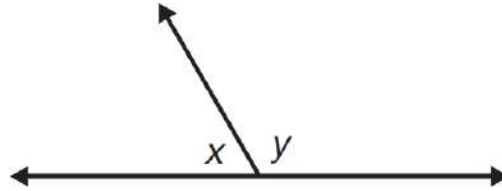
Lines, Angles, and Triangles

SAT Math Strategy

Unless the question states otherwise, the drawings on the SAT math will all be done to scale, so do not hesitate to estimate based on the drawings. The angle rules below are very helpful to know, but if you forget them on test day, making estimates is an excellent backup plan.

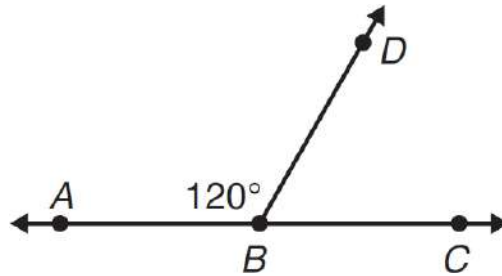
Supplementary Angles

Supplementary angles add up to 180 degrees. The most common example of supplementary angles is when two angles are formed with a straight line across the bottom.



The sum of x and y is 180 degrees.

► Example



Given that $\overline{ABC}$ is a straight line that is intersected at point B by $\overline{BD}$, and that $\angle ABD$ is 120° , what is the measure of $\angle DBC$?

✓ Solution

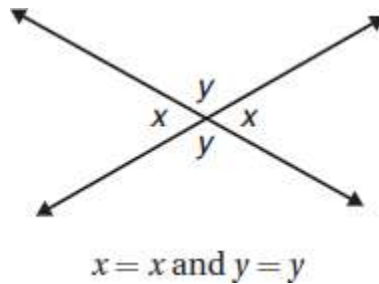
The two angles add up to 180° , so subtract to find the answer, using x as the unknown angle measure:

$$\begin{aligned}x + 120 &= 180 \rightarrow \\x &= 180 - 120 = 60\end{aligned}$$

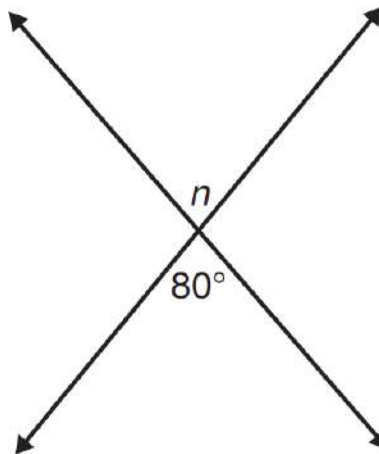
So, $\angle DBC = 60^\circ$

Vertical Angles

When two lines intersect, the angles that are across from one another are called *vertical angles*. Vertical angles are equal to one another.



► Example



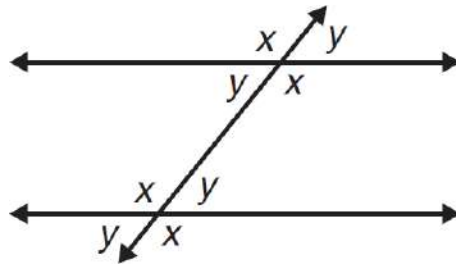
Two lines intersect one another in the figure above. What is the measure of $\angle n$?

✓ Solution

Since $\angle n$ is a vertical angle to the angle of 80° across from it, $\angle n$ will be equal to this angle and will therefore have a measure of 80° .

Parallel Lines and a Transversal

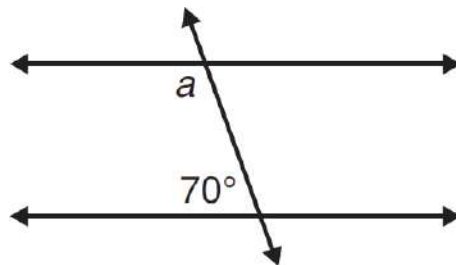
When two parallel lines are cut by a *transversal*, a line that intersects both lines, there are angle equivalencies as labelled below:



It can help to remember that the “big” angles (in this case x) are all equivalent and the “small” angles (in this case y) are all equivalent.

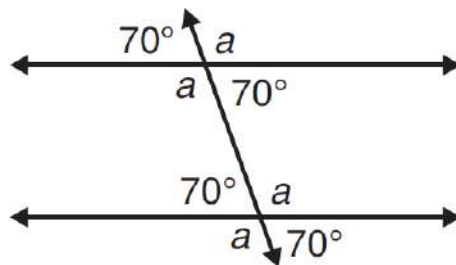
► Example

Two parallel lines have a transversal running through them, as indicated in the drawing below. What is the measure of $\angle a$?



✓ Solution

The measures of all the angles are labelled below:



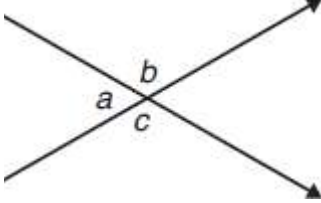
As you can see, the “big” angles are all equivalent and the “small” angles are all equivalent. To calculate the value of a , subtract 70° from 180° since a and 70° are supplementary:

$$180 - 70 = 110$$

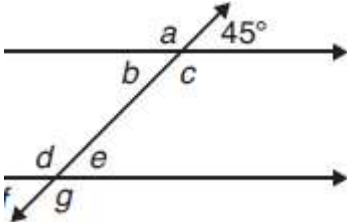
So, a is equal to 110° .

Practice

$\angle x$ is supplementary to an angle of 40 degrees, what is the measure of $\angle x$?



Two lines intersect above, forming angles with measures a , b , and c degrees. If b is twice the measure of $\angle a$, what is the measure of $\angle c$?



Two parallel lines are intersected by a transversal, as given above. What are the measures of the labelled angles?

Solutions

40) Since $\angle x$ is supplementary to 40° , $\angle x$ is equal to 180° minus 40° .

$$180 - 40 = 140$$

20) $\angle b$ and $\angle a$ are supplementary, so they will add up to 180° . Since $\angle b$ is twice the measure of $\angle a$, $b = 2a$. Now, plug $2a$ in for b to solve for the angle measure of a :

$$\begin{aligned} b + a &= 180 \rightarrow \\ (2a) + a &= 180 \rightarrow \\ 3a &= 180 \rightarrow \\ a &= 60 \end{aligned}$$

Since $\angle b$ and $\angle a$ are supplementary, find the value of $\angle b$ by subtracting 60 from 180: $180 - 60 = 120$. $\angle b$ is 120° , and since $\angle c$ and $\angle b$ are vertical angles, $\angle c$ is also equal to 120° .

$\angle a$ is supplementary to 45° , so it has a measure of $180 - 45 = 135$. $\angle b$, $\angle e$, and $\angle f$ are all equal to 45° , given that these are parallel lines with a transversal. $\angle a$, $\angle c$, $\angle d$, and $\angle g$ are all equal to 135° . This is an excellent example of a problem in which you can think it through using the facts that the “big” angles are equal and the “small” angles are equal.

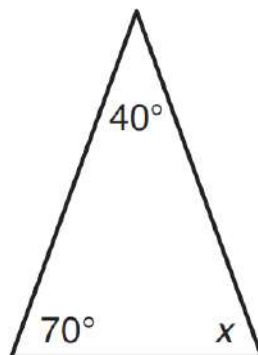
Angles, Triangles, and Quadrilaterals

The SAT gives you this important information on the formula sheet at the beginning:

The sum of the measures in degrees of the angles of a triangle is 180.

Even though this is given, it is extremely helpful to commit this fundamental concept to memory.

► Example



In the triangle above, what is the measure of $\angle x$?

✓ Solution

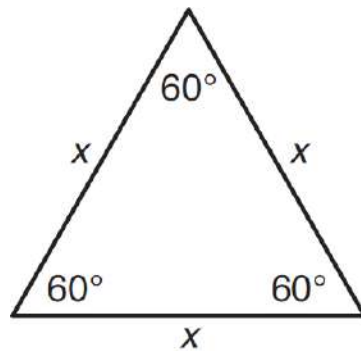
Subtract 70 and 40 from 180 to find the measure of $\angle x$:

$$180 - 40 = 140$$

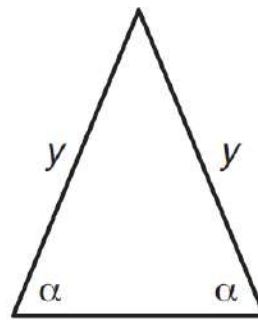
So, $\angle x$ is 70° .

The SAT does NOT give you reminders of these concepts—be sure you memorize them.

- **Equilateral** triangles have all three angles equal (each measuring 60° degrees) and all three sides equal.
- **Isosceles** triangles have only two angles equal and two sides equal.



Equilateral



Isosceles

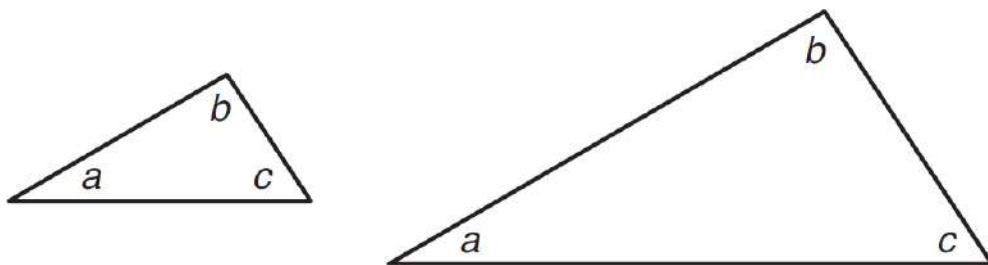
► Example

Is an equilateral triangle also isosceles?

✓ Solution

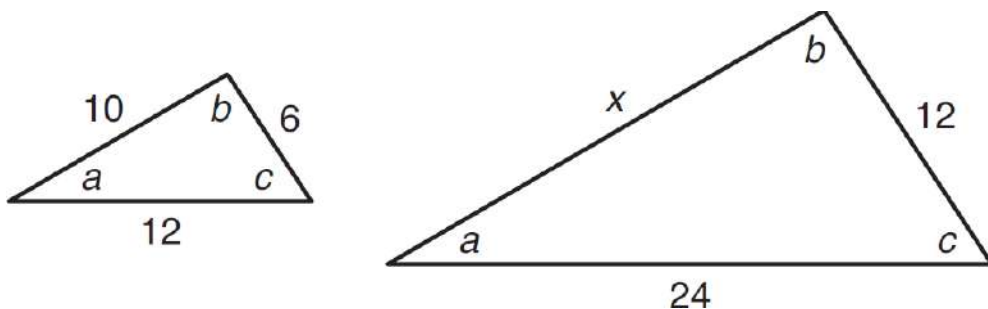
Yes—it is a special form of an isosceles triangle. If two of the angles and corresponding sides are equivalent, the triangle is isosceles. An equilateral triangle does have two angles and sides equivalent. This situation is similar to saying that a square is a special form of a rectangle.

Similar triangles are ones with the same angles, but not necessarily the same sides.



The triangle on the right is larger than the one on the left, yet they are similar—they have the same angle measures and their sides are *proportional* to each other.

► Example



Suppose that the two triangles above are similar to one another. What is the length of side x ?

✓ Solution

Since the triangles are similar, their corresponding sides are proportional:

$$10:6:12$$

$$x:12:24$$

So, set up a proportion to solve for x :

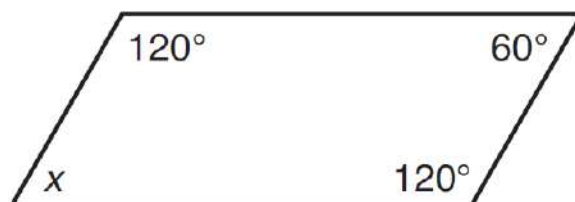
$$\begin{aligned}\frac{x}{12} &= \frac{10}{6} \rightarrow \\ x &= \frac{12 \times 10}{6} = 20\end{aligned}$$

x is therefore 20 units long. You could also have used a different proportion to solve this given the numbers provided.

A final important fact to know about angles is that **a quadrilateral has internal angles that add up to 360 degrees**. This is true for squares, rectangles, parallelograms, trapezoids, and so on.

➤ Example

In the parallelogram below, what is the measure of $\angle x$?



✓ Solution

Subtract the other angles from 360° to find the measure of $\angle x$:

$$360 - 120 - 60 - 120 = 60$$

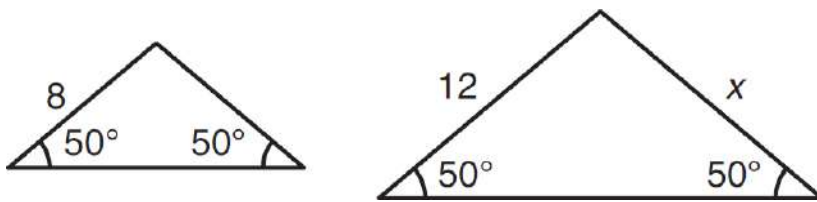
$\angle x$ is equal to the angle opposite it, 60° .

Practice

three of the angles in a trapezoid add up to 300° , what is the measure of the fourth angle?

an isosceles triangle has one angle of 100° , what are the measures of the other two angles?

triangle has two sides with a length of 7 units and two angles that each measure 60° . Based on this information, what is the length of the third side of the triangle?



ased on the drawing of the two triangles above, what is the measure of side ,

Solutions

0°) Since a trapezoid is a quadrilateral, its internal angles will add up to 360°. To find the measure of the fourth angle, subtract 300 from 360:

$$360 - 300 = 60$$

), the fourth angle has a measure of 60°.

0°) A triangle has a total internal degree measure of 180°. Since one of the angles is 100°, it must be the case that the other two angles are equivalent and could add up to 80°. (If the 100° angle were duplicated, the total sum of the internal angles in the isosceles triangle would exceed 200°, which is not possible.) So, subtract 100 from 180 to find the sum of the other two angles:

$$180 - 100 = 80$$

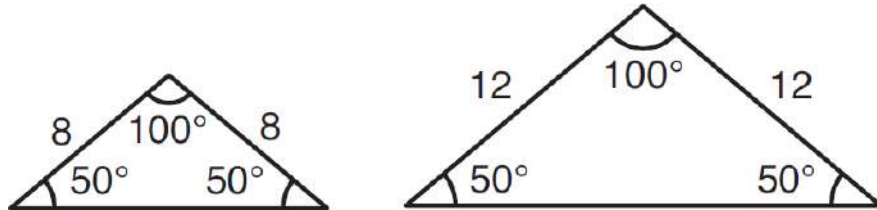
Then, divide 80 by 2 to find the measure of each of the other two angles:

$$\frac{80}{2} = 40$$

), each of the other two angles will measure 40°.

) Since two of the angles in the triangle are each 60°, the measure of the third angle must also be 60° so that the three angles can have a sum of 180°. Since all the angles are equivalent, the triangle must be equilateral. By definition, an equilateral triangle will have equal angles *and* equal sides. Therefore, the measure of the third side of the triangle will be equivalent to the measures of the other two sides—7 units.

2) The two triangles are both isosceles with angle measures of 50° , 50° , and 100° . Since they are similar to one another, the side lengths will be proportional.

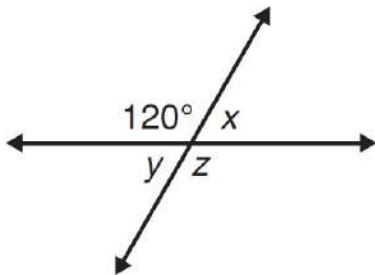


As you can see from the drawings above, the measure of side x will therefore be 12.

Practice Drill—Lines, Angles, and Triangles

(If timing, take about 12 minutes to complete.)

Fill-In Practice: Write your answer in the underlined blank under each question.

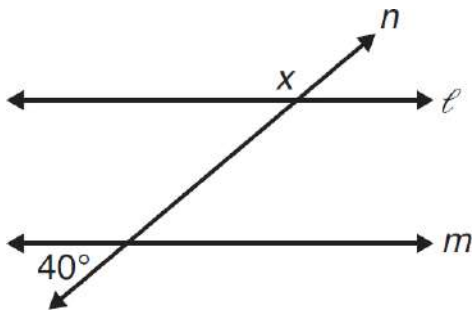


In the drawing above of the two intersecting lines, how many of the angles x , y , and z is/are equal to 120° ?

Answer: _____

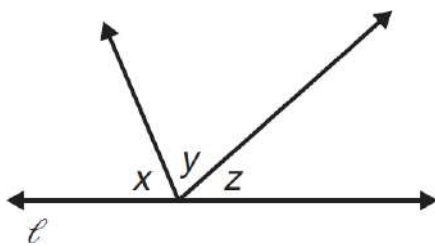
An equilateral triangle has sides of the following lengths in units: 8 , $x - 7$, and $-2x + 38$. What is the value in units of x ?

Answer: _____



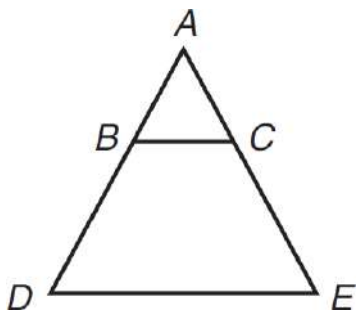
For parallel lines l and m with an intersecting line n , what is the measure of angle x in degrees?

Answer: _____



Given that l is a straight line, what is the mean of the measures of angles x , y , and z in degrees?

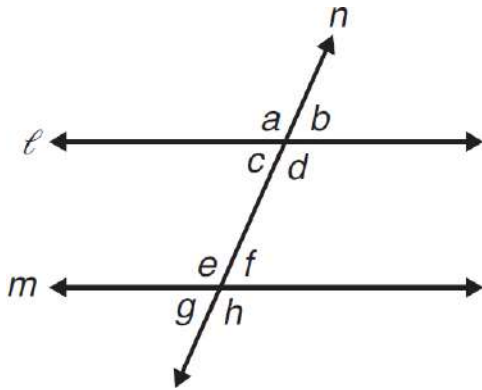
Answer: _____



In $\triangle ADE$, B is a point on $\overline{AD}$ and C is a point on $\overline{AE}$, and $\overline{BC}$ and $\overline{DE}$ are parallel. If $\overline{AE}$ is 12 units, $\overline{AC}$ is 4 units, and $\overline{BC}$ is 3 units, what is the measure of $\overline{DE}$ in units?

Answer: _____

Multiple-Choice Practice



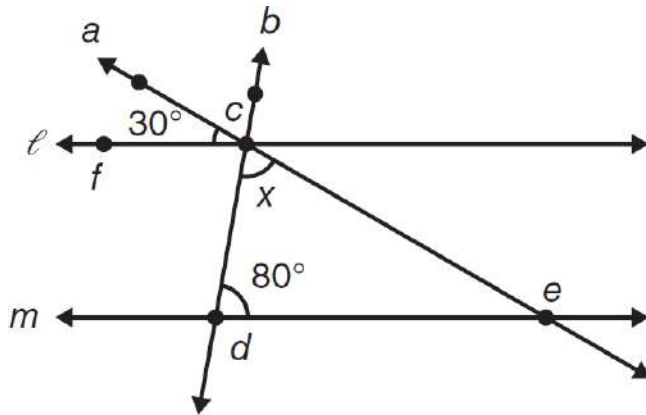
If lines l and m are parallel, and line n intersects both line l and line m , which pairs of angles must be congruent?

- I. $\angle a$ and $\angle h$
 - II. $\angle c$ and $\angle f$
 - III. $\angle e$ and $\angle b$
- A) I only
 - B) III only
 - C) I and II only
 - D) All of the above

If triangle ABC has one angle greater than 90° , what is the maximum number of sides of the triangle that can be equal in length?

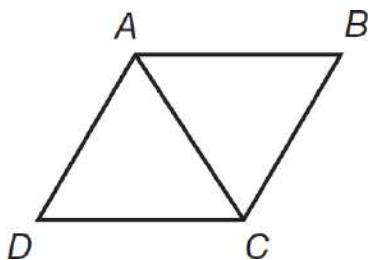
- A) 1

- B) 2
- C) 3
- D) Cannot be determined with the given information



In the above figure, lines l and m are parallel, and lines a and b intersect at point C . If $\angle CDE$ is 80° and $\angle ACF$ is 30° , what is the measure of $\angle x$?

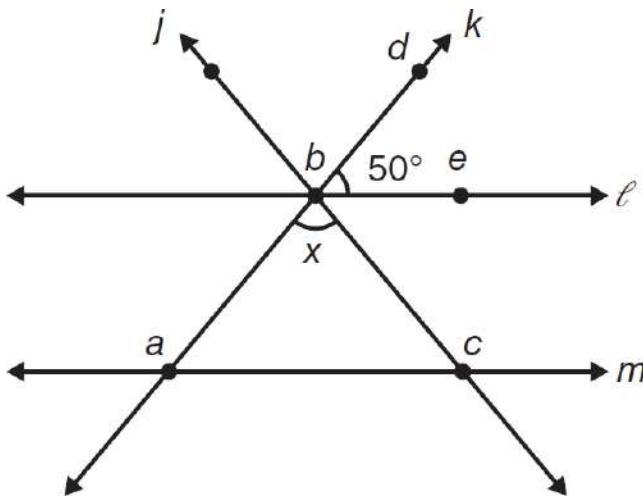
- A) 70°
- B) 80°
- C) 100°
- D) 150°



In the parallelogram $ABCD$ above, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, and $\overline{AC}$ are all congruent. What is the measure of angle $\angle BAC$?

- a) 60°
- b) 80°
- c) 90°

d) 120°



In the graph above, lines l and m are parallel, $\overline{AB}$ and $\overline{BC}$ are congruent, $\angle DBE = 50^\circ$, and lines j and k intersect at point B . What is the measure of angle x ?

- a) 60°
- b) 80°
- c) 100°
- d) 120°

Solutions

1. **(1)** Only the vertical angle that is across from the angle of 120° will also have a measure of 120° . $\angle x$ and the angle across from it each measure 60° .
2. **(15)** All of the sides in an equilateral triangle are equivalent, so set up an equation to solve for x :

$$\begin{aligned}
 8 &= x - 7 \rightarrow \\
 8 + 7 &= x \rightarrow \\
 15 &= x
 \end{aligned}$$

3. **(140)** $\angle x$ is identical to the angle that is supplementary to 40° .
Therefore,

$$\begin{aligned} 40 + x &= 180 \rightarrow \\ x &= 140 \end{aligned}$$

4. **(60)** All three of the angles add up to 180° , so calculate the mean by dividing 180 by 3:

$$\frac{180}{3} = 60$$

5. **(9)** Triangles ABC and ADE are similar to one another because their angles are congruent, so their sides are proportional. Set up a proportion to solve for $\overline{DE}$.

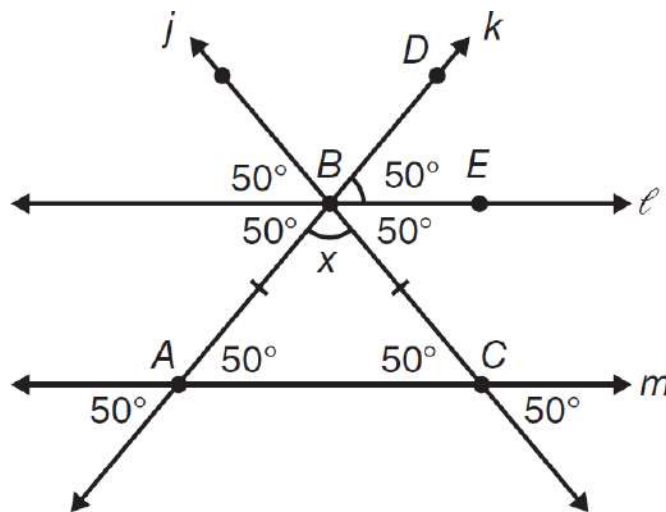
$$\begin{aligned} \frac{\overline{AC}}{\overline{AE}} &= \frac{\overline{BC}}{\overline{DE}} \rightarrow \\ \frac{4}{12} &= \frac{3}{x} \rightarrow \\ x &= \frac{3 \times 12}{4} = \frac{36}{4} = 9 \end{aligned}$$

6. **(C)** With parallel lines and a transversal, match up the “large” and the “small” angles to establish equivalences. $\angle a$ and $\angle h$ are alternate exterior angles that are congruent, and $\angle c$ and $\angle f$ are alternate interior angles that are congruent. $\angle e$ and $\angle b$ are not congruent, which you see from estimation in addition to knowing the general rule.
7. **(B)** It will be impossible for the triangle to have all sides equal, since an equilateral triangle has all angles equal to 60° . However, it is possible to have two of the sides equal since an isosceles triangle could be formed. For example, if the largest angle were 100° and the other two angles were both 40° , the sides opposite the 40° angles would be equivalent.
8. **(A)** Since lines l and m are parallel with a transversal, $\angle CED$ will equal 30° . $\angle x$, $\angle CED$, and $\angle CDE$ add up to 180° because they are within a triangle. Therefore, you can solve for $\angle x$ as follows:

$$x + 80 + 30 = 180 \rightarrow$$

$$x = 180 - 80 - 30 = 70$$

9. **(A)** If all the sides are equivalent, two equilateral triangles, ABC and ACD , will form. Since all the angles in an equilateral triangle are equal to 60° , $\angle BAC$ will equal 60° .
10. **(B)** Use the fact that vertical angles are equal and the angles across from equal sides in an isosceles triangle (i.e., ABC) are equal to establish equivalences as drawn below:

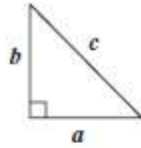


Since $\overline{AB}$ and $\overline{BC}$ are congruent, $\angle BAC = \angle BCA$ (triangle ABC is isosceles). Since lines l and m are parallel and $\angle KBE$ and $\angle BAC$ are corresponding angles, $\angle BAC = 50^\circ$. Therefore, $\angle BCA = 50^\circ$ and $\angle x = 180^\circ - 50^\circ - 50^\circ = 80^\circ$.

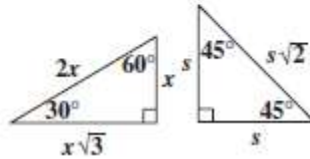
Right Triangles and Trigonometry

Pythagorean Theorem and Special Right Triangles

The SAT provides you with these two important right triangle formulas that enable you to determine the side lengths and even some angles in right triangles (triangles that have a 90° angle):

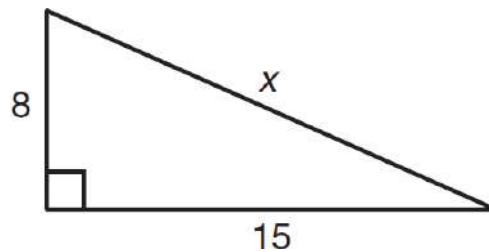


Pythagorean theorem: $a^2 + b^2 = c^2$



Even though they are provided, it is extremely helpful to have these formulas memorized so that you can more instantaneously and intuitively determine right angle relationships.

► Example 1



What is the length of x in the right triangle above?

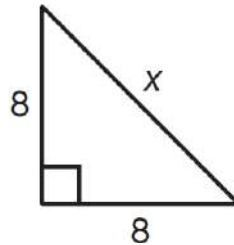
✓ Solution

Use the Pythagorean theorem to solve. Sides a and b are both “legs” of the right triangle (the sides NOT across from the 90° angle), and side c is the hypotenuse (the side that is across from the 90° angle). Let’s use 8 as side a , 15 as side b , and x as side c .

$$\begin{aligned} a^2 + b^2 &= c^2 \rightarrow \\ 8^2 + 15^2 &= x^2 \rightarrow \\ 64 + 225 &= 289 = x^2 \rightarrow \\ \sqrt{289} &= 17 = x \end{aligned}$$

So, side x is equal to 17 units.

➤ Example 2

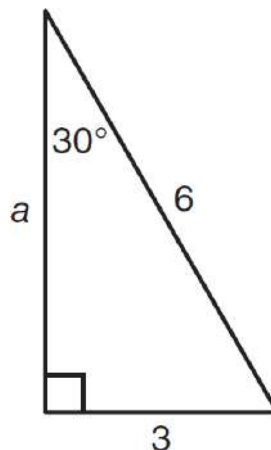


What is the length of side x in the right triangle above?

✓ Solution

While you could use the Pythagorean theorem to solve for x , it will be more efficient to recognize that this is a special right triangle in which the sides have a relationship of $s, s, s\sqrt{2}$. The legs of the triangle are both 8, so you can find the hypotenuse of the triangle by multiplying 8 by $\sqrt{2}$. Therefore, the length of x is $8\sqrt{2}$.

➤ Example 3



What is the length of side a in the right triangle above?

✓ Solution

The above triangle has angles of 90° , 30° , and 60° (since $180 - 30 - 90 = 60$). So, the sides will have a relationship of x , $x\sqrt{3}$, and $2x$. So, instead of having to use the Pythagorean theorem to solve for a , realize that a will simply be $\sqrt{3}$ multiplied by 3, given where the sides are in the triangle. Therefore, $a = 3\sqrt{3}$.

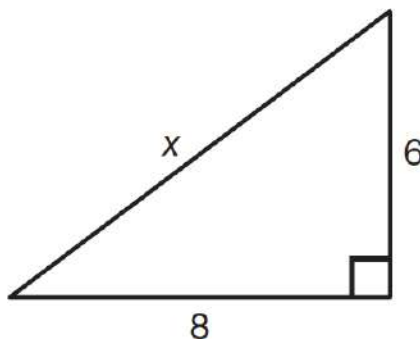
Another way of solving for the unknown sides of right triangles is to use **Pythagorean triples**. These are combinations of side lengths in right triangles for which each value is an integer. The most important ones to know are the following:

3-4-5, 5-12-13, and 7-24-25

Not only will the above combinations work as sides for right triangles, but *multiples* of them will also work. Just as 5-12-13 could be sides of a right triangle, 10-24-26 could also be sides of a right triangle, since each side is twice the value of the sides in the original triple.

► Example

What is the measure of side x in the triangle below?



✓ Solution

While you could solve for x by using the Pythagorean theorem, you can solve this more quickly by recognizing that the sides in this triangle are multiples of a 3-4-5 triple. The 6 is twice 3, the 8 is twice 4, so the x is twice 5. So, to solve for x , just double 5:

$$2 \times 5 = 10$$

Side x therefore has a length of 10 units.

Practice

two legs in a right triangle are 9 and 40, what is the length of the hypotenuse?

the hypotenuse in a right triangle is 61 and one of the legs is 60, what is the length of the other leg?

two legs in a right triangle are equal to y . What is the value of the hypotenuse in terms of y ?

the hypotenuse in a right triangle is equal to 50 and the smaller leg is equal to 14, what is the length of the third side?

the sides in a right triangle are 4, $4\sqrt{3}$, and 8, what is the measure of the smallest angle in the triangle?

Solutions

Use the Pythagorean theorem to solve:

$$\begin{aligned} a^2 + b^2 &= c^2 \rightarrow \\ 9^2 + 40^2 &= c^2 \rightarrow \\ 81 + 1,600 &= 1,681 = c^2 \rightarrow \\ \sqrt{1,681} &= 41 = c \end{aligned}$$

So, the hypotenuse has a length of 41 units.

Use the Pythagorean theorem to solve for the unknown leg. Use 61 as the c and 60 as a :

$$\begin{aligned} 60^2 + b^2 &= 61^2 \rightarrow \\ 3,600 + b^2 &= 3,721 \rightarrow \\ b^2 &= 121 \rightarrow \\ b &= 11 \end{aligned}$$

So, the unknown side has a length of 11 units.

With two of the legs equal in this triangle, we know that the relationship of the sides is $s, s, s\sqrt{2}$. Plugging in y for s , the hypotenuse will be $y\sqrt{2}$.

While you could use the Pythagorean theorem to solve this, you will save time if you recognize that these values are twice what we would find in the Pythagorean triple of 7-24-25. So, double the 24 to find the third side: $24 \times 2 = 48$.

Since the sides have the ratios $x, x\sqrt{3},$ and $2x$ (with $x = 4$), the triangle will have angles of $30^\circ, 60^\circ,$ and 90° . The smallest angle will therefore have a measure of 30° .

Sine, Cosine, Tangent

In a right triangle (a triangle with a 90° angle), the three sides are called the following:

Hypotenuse: This side is across from the 90° angle, and is always the longest side of the triangle.

Opposite: The side that is opposite, or across from, a particular angle. It could be either of the legs of the triangle depending on where the angle is.

Adjacent: The side that is adjacent to, or next to, a particular angle. It also could be either of the legs of the triangle depending on where the angle is and will never be the hypotenuse.

For example, in the right triangle below, the hypotenuse and the sides opposite and adjacent to angle x are labelled:

